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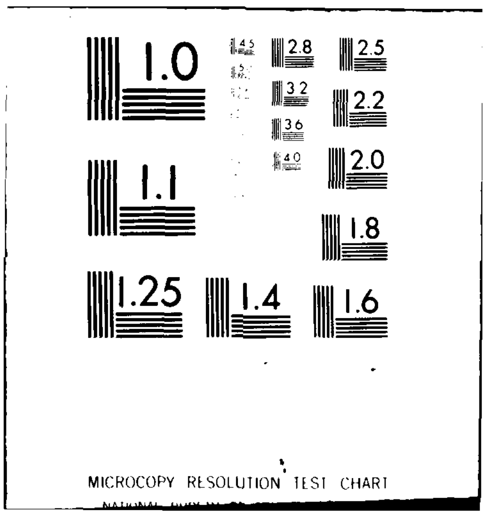
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FOREIGN TECHNOLOGY DIVISION

THEORY OF CODING INFORMATIONAL SIMULATION

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UNEDITED MACHINE TRANSLATION

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PREPARED BY:

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FOREIGN TECHNOLOGY DIVISION
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TABLE OF CONTENTS

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ë in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

Page 1.

THEORY OF CODING INFORMATIONAL SIMULATION.

Page 2.

Collector/collection contains the articles, dedicated to questions of theory and practice of the nonpositional systems of numeration (to deparallelization of the operation of rounding, to the algorithm of detection and correction of errors, to the calculation of elementary functions, to the nomographic method of the image of information), to problems of informational simulation (to optimum layouts of the computer centers in the republic, to the statistical models of exchange systems by information, to the tasks of scheduling, etc.), of organization of computers and systems, and also to use/application of threshold elements/cells in the logic circuits, to combinatory tasks, connected with the development of the algorithms of the compression of information.

Collector/collection can be recommended to the wide circle of the specialists in the region of the theory of coding, computer technology, combinatory methods.

Page 3.

SOME QUESTIONS OF THE STRUCTURE OF SPECIAL-PURPOSE TSVM [DIGITAL COMPUTER].

I. Ya. Akushskiy, V. G. Yevstigneyev.

The contemporary level of the development of microelectronics makes it possible to produce all devices/equipment TsVM, except ZU, in the micro-execution. This means that became possible the construction TsVM from the large/coarse functional boxes, which contain 100-200 logic elements, made in the single technological process. Memory units, both operational, and lasting, are most frequently implemented on the ferromagnetic carriers, pierced by electrical conductors for exciting the carrier and for removing/taking from it the information.

The physical nature of the data carriers, and also the presence of such specific for ZU assemblies as current-operated keys, recording amplifiers and reading, make with its sensitive to different environmental factors, which imply a reduction in its high speed and manufacturability of production. Therefore at present is conducted intense research on an increase in the manufacturability, stability and high speed ZU. These qualities can be achieved/reached, if we make ZU in the single technological process, as far as possible which coincides with the process of manufacture of logic elements TsVM. This leads to the thought of making all blocks TsVM from the elements/cells, carried out on the identical physical basis. Since integral technology makes it possible to obtain the high degree of the integration of semiconductors (transistors and diodes), then we will assume all blocks TsVM integral, semiconductor.

In contemporary aerospace TsVM of those having the rigid, protected in DZU program, the instruction system counts, as a rule, not more than 60 operations; usually their 15-20.

Page 4.

If program consists of M instructions, then each operation in this program is repeated on the average $\frac{M}{2^n} \sim \frac{M}{15 \div 20}$ of times where n - quantity of digits, abstracted/removed in the instruction for the code of operation.

Since the same nuclei OZU can be used for the solution of different problems, then the address of each nucleus OZU is repeated in the program on the average $\frac{M}{2^{m-n}}$ of times where m - discharge/digital configuration of instruction.

Organization DZU of programs.

Let there be the program on M of instructions, then location counter must contain not less than $l=\log_2 M$ of digits. Let us place after counter decoder on M of output combinations. This means that to each instruction of program corresponds one of M outputs/yields. After decoder let us place $N=2^n$ the OR gates whose outputs/yields will correspond to operations, and $L=2^{m-n}$ the OR gates whose outputs/yields let us place in the conformity to the addresses of operands. The first group of OR gates let us name the shaping unit of operations (BFO), the second - by shaping unit of address (BFA). The described structure is depicted in figure 1. From the aforesaid it follows that DZU falls into two parts. The first of them is realized by the structure depicted in figure 1, second, named ZU of constants, either it is included in the composition OZU or it is selected into single DZU of constants.

Evident also that the outputs/yields of blocks FBO and BFA are represented by the single-progression code 1-N and 1-L. If N is small and the organization of its single-progression representation does not cause technical difficulties, then L can reach the value of several thousands; single-progression representation of this value is little attractive due to the unwieldiness. Here we approached a question of organization CZU and DZU of constants.

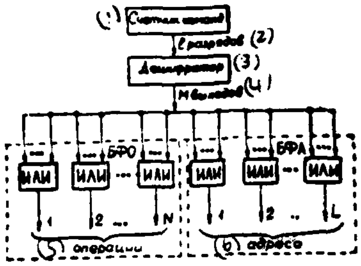

Fig. 1. Block diagram of the formation of operation numbers and addresses.

Key: (1). Location counter. (2). of digits. (3). Decoder. (4). M of outputs/yields. (5). operation. (6). address.

Page 5.

Organization OZU.

The classical diagram of organization of OZU, depicted in figure 2a, provides for, as a rule, two steps/stages of the conversion (deciphering) of address.

Index register OZU (PrA) usually conditionally is divided/marked off into two parts (desirably identical), one of which is designated as index register X(PrAx), another - as index register Y (PrAy). Decoders DShX and DShY are first stage of the conversion of address. Their outputs, represented in single-progression code, are directed into the memory/memorizing matrix/die M, where in the points of the intersection of busbars/tires X and Y are established/installed storage elements; their quantity, equal to X•Y determines capacity OZU. Matrices/dies M it is the second step/stage of the conversion of address.

If in the structure, represented in the figure 2a each address OZU is a conjunction of l variable/alternating, then in the structure in figure 1 each address is the logical function, comprised of different number of disjunctions of conjunctive terms from l of variable/alternating. The address of the nucleus of OZU of structure in figure 2 can be registered in the form

$$A=\overset{1}{\Lambda}a_i=\overbrace{[\overset{1}{\Lambda}a_1a_2\ldots a_{\frac{l}{2}}]}^{Ax}\cdot\overbrace{[\overset{1}{\Lambda}a_{\frac{l}{2}+1}a_{\frac{l}{2}+2}\ldots a_l]}^{Ay}, \qquad (1)$$

where i=1, 2, ..., l.

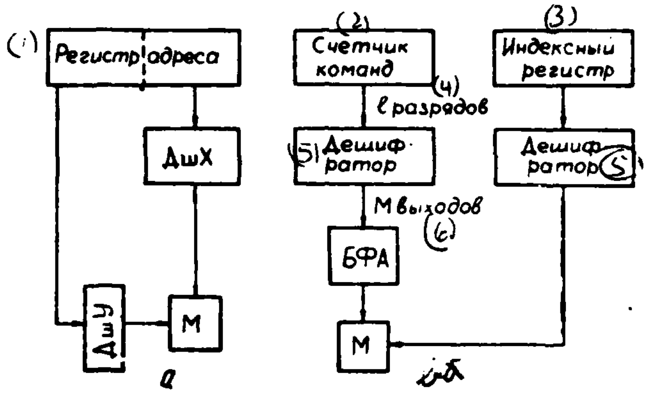

Fig. 2. Block diagram of the formation of addresses of OZU.

Key: (1). Index register. (2). counter of instructions. (3). Index register. (4). digits. (5). Decoder. (6). outputs/yields.

Page 6.

This recording is valid, when numerical length l is even; with l odd

$$A=\overset{1}{\Lambda}a_l=\overbrace{[\overset{1}{\Lambda}a_1a_2\ldots a_{\frac{l-1}{2}}]}^{Ax}\cdot\overbrace{[\overset{1}{\Lambda}a_{\frac{l-1}{2}+1}a_{\frac{l-1}{2}+2}\ldots a_l]}^{Ay}\ldots. \qquad (1')$$

Expression (1) and (1') make legal two-stage deciphering addresses. The address of the nucleus of OZU of structure on figure 1 can be registered in the form

$$A=\underset{1}{\vee}Q_1Q_2\ldots Q_M, \qquad (2)$$

where $Q_k = \overset{1}{\Lambda} a_i,\ i=1,\ 2, \ldots, l, k=1,\ 2, \ldots,\ M.$

A quantity of the conjunctive terms Q lies/rests in the range from k=0 to k=M and depends on the degree of utilization of this nucleus OZU in the program.

From expression (2) it follows that the two-stage deciphering of address in that form in which it is allowed/assumed by expression (1), for the structure in figure 1 is in general not applied. However, sometimes certain similarity of the two-stage deciphering of address to obtain is possible. Thus, during the installation of specialized BTsVM [high-speed digital computer] on to the boat of multiengine aircraft, the program, comprised for one engine, can be repeated for other engines with the appropriate constants, input and output data. For this it is necessary to only have the special (index) register the contents which will be changed per unit after each cycle of the work of program. Entire program they divide/mark off into some quantity of sections and at the end of each of them are changed content of index register per unit. A quantity of sections must be selected by such so that

$$2^{l_{\text{инд. рег.}}} = N_{Ax} = N_{Ay},$$

where $l_{\text{инд. рег.}}$ - quantity of digits of index register;

N_{Ax}, N_{Ay} - quantity of outputs/yields of decoders.

Then the diagram of the formation of address of OZU will take the form, depicted in figure 2b.

Organization DZU of constants.

The classical diagram of organization of DZU is analogous depicted in figure 2a.

One of the versions of the organization DZU of constants for the structure in figure 1 is the version, similar described above for OZU.

Page 7.

In this case is retained the address principle of rotation/access. Since for each task there is its set of constants, than the recurrence of the addresses of constants is very low. It is possible to entirely exclude it, and then will become possible two- or more stepped deciphering of address. But volume of DZU of constant it can prove to be very large; therefore let us consider the possibilities of its decrease.

Each high constant (number) can be presented in the form of the sum of the representatives of the classes of unity, tens, hundreds, etc., i.e., $K=\sum_{i=1}^{n} k_{ij}$, where i=1, 2, ..., n - quantity of classes; j=0, 1, 2, ..., 9 - quantity of constants in each class. For example, a number 3427=3000+400+20+7. It means, instead of one high constant 3427 it is necessary to preserve four small ones and to fulfill four operations of addition. But indeed such low constants, for example, for range P=0-4096 it is necessary to have: unity - 10, ten - 9, hundred - 9, thousand - 4, in all - 32. After decomposing them to four groups (class) of unity, ten, hundred and thousand, we will obtain that for calculating any high constant in the instruction system BTsVM it is necessary to have the special four-address instruction, in which are indicated the addresses of the low constants from each group, which are subject to addition. The capacity DZU of constants for this range becomes constant and with an increase in the latter increases very slowly. The dependence of the capacity DZU of constants on the range takes the form

$$0{,}1\,V_{mzr_k}+\lg V_{mzr_k}=1+\lg(P+1).$$

During this organization DZU of constants, on one hand, is reduced its capacity, on the other hand, appears the need for spending time on the formation/education of high constant. In view of the smallness of the capacity DZU of constants it can be carried out on the semiconductors (for example, on the diode matrices/dies) with the high speed operation.

For further decrease of time for the formation/education of high constant it is necessary to have an adder with the high speed operation, desirably single-cycle. Best for the present instance is the adder, which works in the nonpositional numeration system, for example, in the residual classes.

Let us consider one additional version of the organization DZU of constants. Let it be in the program it used by N of n-bit constants. The shaping unit of the address of constants is diagram with l- input and N- outputs/yields, i.e., (l, N) - pole. DZU itself of constants is diagram with N- input and n- outputs/yields, i.e., (N, n) -pole. Task consists of the replacement of these two (l, N)- and (N, n)-poles one (l, n)-pole.

Page 8.

If we examine the task of synthesis separately for each of the n digits, then we will have n(l, 1)-poles to each of which will be distinctive maximum N/2 disjunctions. But if we for this diagram use the methods of the synthesis (l, n)-poles than possible in certain cases it will substantially simplify logic circuit.

Let us name this method of organizing DZU the constants no-address.

Organization for transfer of control.

The instructions of transfer of control (conditional and unconditional transfers) occupy in any program the considerable place. The special feature/peculiarity of these instructions lies in the fact that they act not on the operands, but on location counter, changing the natural order of its work.

The code of the operation of transfer of control, formed with the shaping unit of operation of BFC in figure 1, determines the moment/torque when contents of location counter must be changed to the larger or smaller side. Value and direction of change are assigned by the code, which stands in the address part of the instruction. Let us consider the different methods of changing contained location counter.

Figure 3a, depicts the most widely used method of changing contained location counter when to the register of transition is brought in the code, registered in the address part of the instruction, and from the latter it is rewrited into location counter. In this counter circuit besides the calculating must have even and adjusting input. A quantity of digits of the register of transition must be equal to a quantity of discharges of location counter.

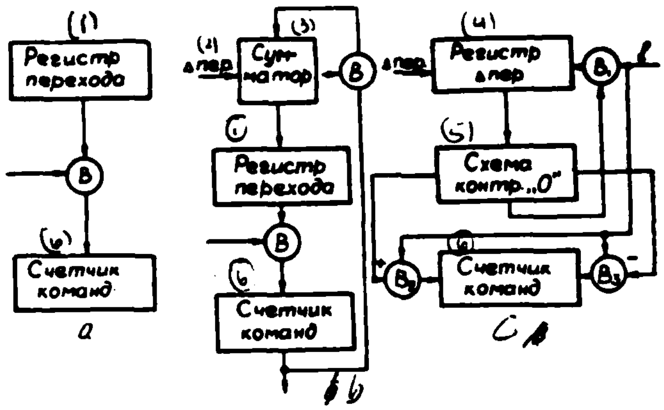

Fig. 3. Schematic of a change in contained location counter.

Key: (1). Register of transition. (2). per. (3). Adder. (4). Register. per. (5). Diagram cont., "0". (6). Location counter.

Page 9.

If diagram for the formation of each jump operation is (l, 1)-pole then the diagram of the formation of the code of transition consists of q(l, l)-poles where q - quantity of different instructions of the transfer of control (for example, if we in the instruction system eat instructions BP, UP(+), UP(-), UP(0), then q=4); l - quantity of discharges the register of transition.

Figure 3b depicts another version of the schematic of a change in contained location counter. Here the code on the register of transition is formed by the algebraic addition of contained location counter and value of a change in the code of the latter (step/pitch of transition), i.e.,

$$\Delta_{\text{пер}} = \varepsilon_{\text{пер}} - \varepsilon_{\text{сч.к.}}$$

On the register of transition we have

$$\varepsilon_{\text{pr.пер}} = \varepsilon_{\text{сч.к}} + \Delta_{\text{пер}} = \varepsilon_{\text{пер}}.$$

Transition from the formation of the full/total/complete code of transition $\varepsilon_{\text{пер}}$ to the formation of the code of the step/pitch of transition allows from the diagram, which contains $q(l, l)$-poles, to switch over to the diagram, which contains $q(l, \mu)$-poles where μ - quantity of bits of code $\Delta_{\text{пер}}$, which can be done considerably smaller l.

A deficiency/lack in this diagram is the presence of adder; however, its functions can be in certain cases transmitted to the adder of arithmetic unit.

The modification of the described diagram is the diagram, depicted in figure 3c.

The step/pitch of transition, formed with the appropriate diagram, with sign (+) or (-) will be brought in in $\text{Pr}\Delta_{\text{пер}}$. This register must be carried out in the form of counter or shift register. The diagram of check of zero opens/discloses valve B_1 and one of the valves B_2 or B_3. Reference frequency f_{on}, entering in $Pr\Delta_{nep}$, reduces its contents to 0. This moment/torque is monitored by the diagram of check which closes the open valves. The quantity of impulses/momenta/pulses, spent on resetting to zero $Pr\Delta_{nep}$, was carried to the bidirectional counter of instructions the contents which was changed to value Δ_{nep}. Frequency f_{on} can be selected sufficiently high, and bit configuration $Pr\Delta_{nep}$ - it is sufficient low. But if value Δ_{nep} - is greater than can contain $Pr\Delta_{nep}$, then in the program it is possible to place two or more instructions of transfer of control in a row then so that the total code of transition would be equal to

$$\Delta_1+\Delta_2+\ldots+\Delta_n=\Delta_{nep}.$$

Page 10.

Since value Δ_{nep} carries random character, then the optimum bit configuration of register $Pr\Delta_{nep}$, probably, will be equal to $\log_2$ the mathematical expectation of all values Δ_{ni} of this program, i.e.,

$$\mu_{onr}=\log_2 M[\Delta_{nep}].$$

Finally the bit of register $Pr\Delta_{nep}$ should be chosen only after the analysis of concrete/specific/actual program taking into account of high speed and equipment expenditures.

On the basis of entire of that presented it is possible to propose structure TsVM with the wired program (Fig. 4).

The work of TsVM indicated occurs thus.

1. Under interaction of cadence generator contents of counter of commands increases per unit.

2. Shaping unit of address BFA, carried out in the form of logic circuit, forms/shapes addresses of nuclei OZU in single-progression code. Let us name their address Y.

From the output/yield of the register of indices is formed/shaped single-progression address OZU (address X).

The shaping unit of constants BFK, carried out in the form of logic circuit, forms/shapes constant, if the same is necessary on this stroke/cycle of the work of location counter.

The shaping unit of operations BFO, carried out in the form of logic circuit, forms/shapes the single-progression code of operations; to each operation corresponds single output/yield.

3. If is formed address of nucleus of OZU, then is produced reading of operand; if there is no same, then is used formed

constant.

4. If is formed arithmetic or sending operation, then arithmetic unit implements it.

If is formed the operation of transfer of control, then the shaping unit of the transition BFP together with location counter implements it.

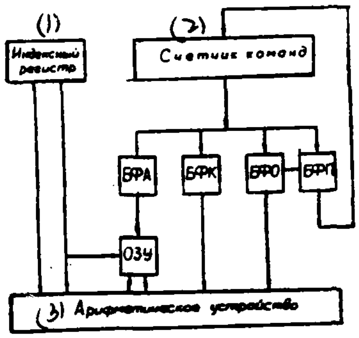

Fig. 4. Block diagram of TsVM with the wired program.

Key: (1). Index register. (2). Location counter. (3). Arithmetic unit.

Page 11.

From the description it is evident that with the work of this TsVM is not implemented one traditional stroke/cycle - reading of instruction of DZU, what it was possible to achieve, after replacing this process with the set of logical blocks.

Questions of the equipment realization of TsVM of this structure represent the object/subject of further experiments, since it is closely related to the specific problems and the programs.

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Page 12.

Of construction of nomograms in numeration system in the residual classes (SSOK).

I. Ya. Akushskiy, A. Ya. B'yanzin.

The nomographic methods of the solution of problems possess large simplicity and clarity. Nomograms are compact, and response/answer through them is located rapidly. However, the accuracy of the solution is limited by the sizes/dimensions of drawing and, as a rule, it is low.

In work [1] for the purpose of an increase in the accuracy is done the attempt to consider a question of the practical use/application of a system of nomograms in SSOK for the solution of some problems. However, in it are illuminated the particular sides of this question and the proposed technical realization is limited to nomograms with the rectilinear scales.

Work [2] examines the possibility of designing of device/equipment, capable of using with the nomograms, which have arbitrary scale shape. The proposed in works [1] and [2] solutions assume the need of storing the periodic answer scale, which introduces the considerable unjustified redundancy. Furthermore, it is difficult to use nomograms for the reverse operations as a result of the ambiguity of the marks of the answer scale.

In work [3] these deficiencies/lacks are somewhat reduced, but they are not eliminated. This is connected with the fact that during the construction of nomograms on the independent foundations and during the determination of response/answer on them is used the positional representation (image) both scale of nomograms and resolving straight lines.

Transition to the nonpositional representation of the scales of nomograms and resolving straight lines will make it possible to reduce the deficiencies/lacks indicated and to obtain the qualitatively new properties of nomograms.

This work is dedicated to questions of construction and research of different functional dependences taking into account the specific character of deductions concerning modulus/module p, since precisely these questions play principal role in construction and use of systems of nomograms SSOK.

Page 13.

For this let us introduce some concepts and we trace the properties of the functions, examined on the bounded set of integers.

Plane of integers on modulus/module p $(|\Pi_Z|_p)$

The locus, that corresponds to many regulated pairs of numbers in set $|Z\times Z|_p$, where $|Z\times Z|_p = |Z|_p \times |Z|_p$, Z the set of integers, $|Z|_p$ - the set of the integers, undertaken on modulus/module p; let us name its plane of integers on modulus/module p and let us designate $|\Pi_Z|_p$.

Since $|Z\times Z|_p \subset Z\times Z$, to plane $|\Pi_Z|_p$ corresponds only the part of the points, which lie on the plane of integers Π_Z.

Let us point out some properties of plane $|\Pi_Z|_p$.

1. Plane $|\Pi_Z|_p$ is limited, since it contains pXp points.

Consequently, all points of plane $|\Pi_Z|_p$ lie/rest squared by size/dimension (p-1) (p-1).

2. Plane $|\Pi_Z|_p$ contains only points with positive integer coordinates x and y and is representation of entire plane Π_Z, moreover

any point of plane Π_Z unambiguously is mapped into point, which lies on plane $|\Pi_Z|_p$, and to each point of plane $|\Pi_Z|_p$ corresponds countless number of points of plane Π_Z.

Straight line on plane $|\Pi_Z|_p$

Straight line on plane $|\Pi_Z|_p$ we will call locus, which satisfy the equation of the form

$$|Ax+By+C|_p=0, \tag{1}$$

where x, y, A, B, C, $\in |Z|_p$.

Since plane $|\Pi_Z|_p$ is the representation of entire plane Π_Z, of straight line $|Ax+By+C|_p=0$ on plane Π_Z it corresponds the family of the straight lines of the form

$$\tilde{A}\tilde{x}+\tilde{B}\tilde{y}+\tilde{C}=0,$$

where $A=|\tilde{A}|_p$, $B=|\tilde{B}|_p$, $C=|\tilde{C}|_p$, $x=|\tilde{x}|_p$, $y=|\tilde{y}|_p$,
$\tilde{A}\equiv A(\mathrm{mod}\ p)$, $\tilde{B}\equiv B(\mathrm{mod}\ p)$, $\tilde{C}\equiv C(\mathrm{mod}\ p)$, $\tilde{x}\equiv x(\mathrm{mod}\ p)$,
$\tilde{y}\equiv y(\mathrm{mod}\ p)$.

Page 14.

By straight/direct on plane Π_Z we understand the point set of the straight line Ax+By+C=0 with the integral coordinates.

On plane $|\Pi_Z|_p$ of straight line with the angular ones coefficient k let us name the point set of plane $|\Pi_Z|_p$, of coordinate of which they satisfy the equation of the form

$$y = |kx+b|_p, \qquad (2)$$

where k, b, y, $x \in |Z|_p$.

Straight line (2) on plane Π_Z corresponds the family of the straight lines of the form

$$\tilde{y} = \tilde{k}\tilde{x} + \tilde{b},$$

where $\tilde{y} \equiv y(\mathrm{mod}\ p)$, $\tilde{x} \equiv x(\mathrm{mod}\ p)$, $\tilde{k} \equiv k(\mathrm{mod}\ p)$, $\tilde{b} \equiv b(\mathrm{mod}\ p)$.

The equation of the form

$$y = |kx|_p \qquad (3)$$

determines the straight line, passing through the origin of coordinates.

Let us point out some properties of operations with the smallest positive remainders/residues, which escape/ensue from the properties of comparisons [4] and necessary for the following presentation.

1. $|a \pm b|_p = ||a|_p \pm |b|_p|_p$.

2. $|a\cdot b|_p=||a|_p\cdot|b|_p|_p$.

3. $\left|\frac{a}{b}\right|_p=\left|\frac{|a|_p}{|b|_p}\right|_p$.

4. $|a|_p=a$, if $0\leq a<p$.

5. If $|a-b|_p=0$ and $0<a<p$, $0<b<p$, then $a-b=0$.

6. If $|a-b|_p=0$, then $|a|_p-|b|_p=0$.

7. If $|a\cdot b|_p=0$, moreover $0\leq a<p$, $0<b<p$ and p - prime number, then either $a=0$ or $b=0$.

8. If $|a\cdot b|_p=0$, then $|a|_p\cdot|b|_p=0$.

9. If $\left|\frac{a}{b}\right|_p=0$ and $0\leq a<p$, $0<b<p$, then $a=0$.

10. If $\left|\frac{a}{b}\right|_p=0$, then $|a|_p=0$.

11. If $a=0$, then $|a|_p=0$.

12. If $|a+b|_p=0$ and $0\leq a<p$, $0\leq b<p$, then either $a=0$ and $b=0$ or $a+b=p$.

If the absolute value of number a is lower than p, then $|-a|_p=p-a$.

Page 15.

Properties of straight lines on plane $|\Pi_z|_p$

Through any two points of plane $|\Pi_z|_p$ it is possible to draw straight line and besides only one.

Proof. Let on plane $|\Pi_z|_p$ be given two arbitrary points $A(x_1, y_1)$ and $B(x_2, y_2)$. Through these points on Cartesian plane Π we carry out straight line; this is possible in view of the axiom of geometry, such straight/direct single. Its equation takes the form

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}, \text{ если}^{(1)}\ x_2\neq x_1,\ y_2\neq y_1$$

Key: (1). if.

or

$$y=kx+b,$$

where

$$k=\frac{y_2-y_1}{x_2-x_1},\ b=\frac{y_1x_2-y_2x_1}{x_2-x_1}.$$

If $x_2=x_1$ or $y_2=y_1$, then the equation of straight line will respectively take form $x=x_1$ or $y=y_1$.

During the construction by straight line on plane $|\Pi_z|_p$ are possible two cases.

1. $k=\frac{y_2-y_1}{x_2-x_1}$ integer, smaller p.

Then to the straight line y=kx+b on plane Π_z, passing through two points A and B, corresponds single straight/direct $y=|kx+b|_p$, passing through the same points on plane $|\Pi_z|_p$, since any point of plane Π_z unambiguously is mapped into point on plane $|\Pi_z|_p$.

2. $k=y_2-y_1/x_2-x_1$ - not integer.

We carry out direct

$$y=\left|\frac{y_2-y_1}{x_2-x_1}\right|_p x+\left|\frac{x_2y_1-x_1y_2}{x_2-x_1}\right|_p$$

on plane Π_z, angular coefficient of which $\left|\frac{y_2-y_1}{x_2-x_1}\right|_p$ - whole, obtained as a result of the formal division y_2-y_1 on x_2-x_1 on modulus/module p. To it corresponds the unique straight line

$$y=\left|\left|\frac{y_2-y_1}{x_2-x_1}\right|_p x+\left|\frac{x_2y_1-x_1y_2}{x_2-x_1}\right|_p\right|_p$$

on plane $|\Pi_z|_p$.

Page 16.

Let us show that points $A(x_1, y_1)$ and $B(x_2, y_2)$ lie/rest on this straight line. (Actually/really on the basis of properties 1-11).

$$y=\left|\left|\frac{y_2-y_1}{x_2-x_1}\right|_p x_1+\left|\frac{x_2y_1-x_1y_2}{x_2-x_1}\right|_p\right|_p=\left|\frac{y_1(x_2-x_1)}{x_2-x_1}\right|_p=|y_1|_p=y_1,$$

since $y_1<p$.

Is analogous

$$y=\left|\frac{y_2-y_1}{x_2-x_1}x_2+\frac{x_2y_1-x_1y_2}{x_2-x_1}\right|_p=y_2,$$

since $y_2>p$.

Let us show that line $y=\left|\left|\frac{y_2-y_1}{x_2-x_1}\right|_p x+\left|\frac{x_2y_1-x_1y_2}{x_2-x_1}\right|_p\right|_p$ - is unique. Actually/really let us assume that there is another straight line $y=|k_1x+b_1|_p$, which passes through points $A(x_1, y_1)$ and $B(x_2, y_2)$. Let us determine k_1 and b_1 from the condition that the coordinates of points A and B satisfy the equation of this straight line, i.e.,

$$y_1=|k_1x_1+b_1|_p,$$
$$y_2=|k_2x_2+b_2|_p.$$

Subtracting y_1 from y_2, we will obtain on the basis of properties A

$$y_2-y_1=|k_1(x_2-x_1)|_p.$$

Since y_2 and y_1 is less than p,

$$|y_2-y_1|_p-|k_1(x_2-x_1)|_p=0$$

or

$$|y_2-y_1-k_1(x_2-x_1)|_p=\left|\frac{y_2-y_1}{x_2-x_1}-k_1\right|_p\cdot|x_2-x_1|_p=0$$

with $x_2-x_1\neq 0$.

Since

$$|x_2-x_1|_p \neq 0,$$

that

$$\left|\frac{y_2-y_1}{x_2-x_1}-k_1\right|_p=\left|\frac{y_2-y_1}{x_2-x_1}\right|_p-|k_1|_p=0,$$

hence

$$|k_1|_p=k_1=\left|\frac{y_2-y_1}{x_2-x_1}\right|_p.$$

Page 17.

From expression $y_1=|k_1x_1+b_1|_p$ let us find b_1.

$$y_1-|k_1x_1+b_1|_p=|y_1|_p-|k_1x_1+b_1|_p=|y_1-k_1x_1-b_1|_p=$$
$$=||y_1-k_1x_1|_p-|b_1|_p|_p=0.$$

Since $|y_1-k_1x_1|_p<p$ and $|b_1|_p<p$, that

$$|y_1-k_1x_1|_p-|b_1|_p=0,$$

whence

$$|b_1|_p=b_1=|y_1-k_1x_1|_p=\left|y_1-\frac{y_2-y_1}{x_2-x_1}x_1\right|_p=\left|\frac{y_1x_2-x_1y_2}{x_2-x_1}\right|_p.$$

Consequently, the straight line, passing through two given points, is unique, since $k_1=k$, $b_1=b$.

Level (3) with different k=0, 1, ..., p-1 determines on plane $|\Pi z|_p$ pencil of straight lines with the center in the beginning of coordinates.

Theorem 1. Through any point of plane $|\Pi z|_p$ passes either the axis of ordinates $x=0, x\in|Z|_p$, or certain straight line of bundle with the center in the beginning cf coordinates.

Proof. Since through two pcints of plane $|\Pi_z|_p$ it is possible to draw straight line and besides cnly one, then two lines of bundle do not have common points, except the crigin of coordinates. Bundle contains p of lines (since k=0, 1, 2 ..., p-1), and each line of the bundle contains p-1 points, except the origin of coordinates.

Consequently, entire pencil cf straight lines contains $p(p-1)+1=p^2-p+1$ cf points. If cne considers that the axis of ordinates has p-1 points (excluding point (0, 0))), then in all it will be $p^2-p+1+p=1=p^2$ points. But this is the total number of points of plane $|\Pi_z|_p$ that also proves thecrem.

Mutual location of points and straight lines cn plane $|\Pi_z|_p$

The point of intersection of two straight lines we will call the common point, which belongs simultaneously to both straight lines.

Straight lines on plane $|\Pi_z|_p$ thcse not having common points, let us name parallel lines.

Lemma 1. Two ncncoincident straight lines on plane $|\Pi_z|_p$ can either intersect only at cne point cr not to intersect generally.

Page 18.

Let the straight lines intersect in two or more points. Consequently, through two points it is possible to draw two different straight lines which contradicts the property of straight lines.

In order to find the point of intersection of two straight lines, let us solve system of equations:

$$\begin{cases} y = |k_1 x + b_1|_p, & (8) \\ y = |k_2 x + b_2|_p. & (9) \end{cases}$$

The coordinates of the point of intersection of straight lines (8) and (9) will take the form

$$\left.\begin{aligned} x &= \left|\frac{b_1 - b_2}{k_2 - k_1}\right|_p \\ y &= \left|\frac{k_2 b_1 - k_1 b_2}{k_2 - k_1}\right|_p \end{aligned}\right\}. \qquad (10)$$

Straight/direct (8) and (9), not having point of intersection, let us name parallel. In this case the condition of the parallelism of straight lines will take the form

$$k_2 = k_1,\ b_2 \neq b_1. \qquad (11)$$

Straight lines (8) and (9) let us name perpendicular, if

$$k_2 = p - \left|\frac{1}{k_1}\right|_p. \qquad (12)$$

If the equations of straight lines are registered in the form

$$\begin{cases} |A_1x+B_1y+C_1|_p=0 & (13) \\ |A_2x+B_2y+C_2|_p=0, & (14) \end{cases}$$

then, by solving system of equations (13), (14), let us find the coordinates of the point of intersection of the straight lines

$$x=\left|\frac{B_2C_1-B_1C_2}{A_2B_1-A_1B_2}\right|_p, \quad y=\left|\frac{A_1C_2-A_2C_1}{A_2B_1-A_1B_2}\right|_p. \qquad (15)$$

In this case the condition of the parallelism of straight lines (13) and (14) will be registered as

$$|A_2B_1-A_1B_2|_p=0, \qquad (16)$$

and the condition of the perpendicularity of straight lines (13) and (14) it will be registered as

$$|A_1A_2+B_1B_2|_p=0. \qquad (17)$$

Page 19.

In the case when three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ lie/rest on one straight line, must be satisfied the condition

$$\left|\frac{y_3-y_1}{y_2-y_1}\right|_p=\left|\frac{x_3-x_1}{x_2-x_1}\right|_p. \qquad (18)$$

Using properties A, condition (18) it is possible to reduce to the form

$$\left\| \begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix} \right\|_p = 0. \qquad (19)$$

Connection between the straight lines on planes $|\Pi z|_p$ and Π

Theorem 2. Straight line $y=|kx+b|_p$ on plane $|\Pi_z|_p$ can be decomposed into k or k+1 sections in such a way that the points of straight line for each of these sections will belong to the straight/direct $y=kx+b_i (i=1, 2, \ldots, k)$ on plane Π, parallel ones between themselves.

Proof. Let us consider straight line (2) on plane $|\Pi_z|_p$.

Let us find the first interval for x in which $0 \leqslant kx+b<p$. In this case of x it will vary from 0 to $\frac{p-b}{k}$. Since $|kx+b|_p=kx+b$, that the points of straight line (2) on plane Π at interval $0 \leqslant x<\left[\frac{p-b}{k}\right]$ lie/rest on straight line $y=kx+b_1$, examined in the interval of the variation x from 0 to $\frac{p-b}{k}$, in this case $b_1=b$.

Let us further find the second interval for x in which $p \leqslant kx+b<2p$. In this case it is obtained, that x must be within the limits from $\frac{p-b}{k}$ to $\frac{2p-b}{k}$. Consequently, in interval $\left[\frac{p-b}{k}\right]<x<\left[\frac{2p-b}{k}\right]$

of the point of straight line (2) on plane Π they belong to straight line $y=kx+b_2$ (where $b_2=b-p$), examined in interval of $\frac{p-b}{k} \leqslant x < \frac{2p-b}{k}$.

Page 20.

Let us consider the i interval for x in which

$$(i-1)_p \leqslant kx+b<ip.$$

In this case of x varies from $\frac{(i-1)p-b}{k}$ to $\frac{ip-b}{k}$. Consequently, in the interval

$$\left[\frac{(i-1)p-b}{k}\right] \leqslant x < \left[\frac{ip-b}{k}\right]$$

of the point of straight line (2) on plane Π belong to straight line $y=kx+b_i$ (where $b_i=b-(i-1)\cdot p$), examined in the interval

$$\frac{(i-1)p-b}{k} \leqslant x < \frac{ip-b}{k}.$$

Let us consider, in what interval falls point x=p-1 at the maximum value of b=p-1. In this case

$$kx+b=k(p-1)+(p-1)=pk+(p-1)-k \geqslant pk,$$

since $k \leq p-1$.

Consequently, $pk \leq kx+b$, and point x=p-1 lies/rests at k+1 interval.

If b=0,

$$kx+b=k(p-1)>kp-p=p(k-1),$$

i.e. point x=p-1 lies/rests at a k-interval. Thus, we have a separation of straight line $y=|kx+b|_p$ in the sections so that the points of straight line for each of these sections belong to one of the parallel between themselves straight lines of family $y=kx+b_i$ on plane Π.

Let us find the points of intersection of straight lines with X and Y axes on plane Π. The first straight line will cross Y axis at point (0, b), the second - at point (0, b-p), the i-th - at point (0, b-(i-1) p).

The distance between (i-1) and the i point is equal

$$[b-(i-2)p]-[b-(i-1)p]=p, \ i=2, 3\ldots$$

Thus, all obtained parallel lines transverse axis Y through the equal gaps/intervals with a length of p.

Since the straight lines are parallel and transverse axis Y through the equal gaps/intervals, then, therefore, they must intersect and X axis also through are equal gaps/intervals.

Let us find the distances between the points of intersection of straight lines with X axis. For this it suffices to determine distance along the axis X among the first and second straight line. The first straight line $y=kx+b_1$ transverse axis X at point $\left(0, -\frac{b}{k}\right)$, and the second straight line $y=kx+b_2$ transverse axis X at point (0, p-b/k). The distance between the points of intersection of the parallel lines in question with X axis is equal

$$\frac{p-b}{k}-\left(-\frac{b}{k}\right)=\frac{p}{k}.$$

Page 21.

Corollary. Any point of straight line $y=|kx+b|_p$ on plane $|\Pi_Z|_p$ lies/rests to one of the straight lines of family $y=kx+b_i$, i=1, 2, ..., k, k+1 on plane Π.

Functions their graphs.

We will consider that if to each element/cell $x\in|Z|_p$ are set in the conformity one or several elements/cells y of set $|Z|_p$, then on set $|Z|_p$ is determined function $y=F(x)$. . By domain of definition and the range of values of function y=F(x) is set $|Z|_p$.

Let us consider the functions, determined on set $|Z|_p$, corresponding to some whole-valued functions, determined on set Z. Thus, if on set Z is determined certain whole-valued function y=f(x), or ϕ(x, y)kp, k=0, 1..., then the corresponding to it function, determined on set $|Z|_p$ we will call the function

$$y=F(|x|_p)=|f(x)|_p \qquad (20)$$

in the case of an explicit assignment of the function and

$$\Phi(|x|_p/|y|_p)=|\varphi(x, y)|_p=0 \tag{21}$$

in the case of the implicit assignment to function.

If on set Z is preset whole-valued function in the parametric form $x=\phi(t)$, $y=\psi(t)$, $t_0\leq t\leq t_1$, then the corresponding to it function on set $|Z|_p$ will be determined then:

$$x=\Phi(|t|_p)=|\varphi(t)|_p,\quad y=\Psi(|t|_p)=|\psi(t)|_p. \tag{22}$$

Let us note that for any whole-valued function, preset on set Z, it is possible to unquiely plot the function corresponding to it on set $|Z|_p$, but to each function on set $|Z|_p$ can correspond the whole family of the functions, determined on set Z.

Plotted function (20), (21), (22) will be the set of the points of plane $|\Pi_z|_p$, of coordinate of which they satisfy relationships/ratios (20), (21), (22).

Page 22.

Functions $y=F(|x|_p)$ let us name monotone not decreasing, if with $|x_1|_p<|x_2|_p$

$$F(|x_1|_p)\leqslant F(|x_2|_p), \qquad (23)$$

and monotone that not increasing, if with $|x_1|_p<|x_2|_p$

$$F(|x_1|_p)\geqslant F(|x_2|_p). \qquad (24)$$

If in expressions (23) and (24) stand absolute inequalities, then function (23) is called monotone increasing, and function (24) - monotone decreasing.

For example, function $y=|x^5|_{11}$, the graph by which is depicted in figure 1, in sections [0, 2], [9, 10] - monotonically increasing, but in section [3, 8] - not monotonically decreasing.

Function $y=F(|x|_p)$ we will call periodic with the period l, where $l\in|Z|_p$, if occurs the equality the form

$$F(||x|_p+l|_p)=F(|x|_p) \qquad (25)$$

for any value $x\in Z$.

For example, function $y=|4^x|_{11}$ - periodic, with the period $l=5$, which is evident from its graph, represented in figure 2.

Function $y=F(|x|_p)$ let us name symmetrical relative to axis x=a, where $a\in|Z|_p$, if

$$F(|a+|x|_p|_p)=F(|a-|x|_p|_p) \tag{26}$$

for all values $x\in Z$. In this case x=a - axis of symmetry. Function $y=F(|x|_p)$ let us name symmetrical relative to point A(a, b), where $a, b\in|Z|_p$, if

$$|b+p-F(|a+p-|x|_p|_p)|_p=|p-b+F(|a+|x|_p|_p)|_p \tag{27}$$

for all values $x\in Z$. Point A(a, b) - center of symmetry.

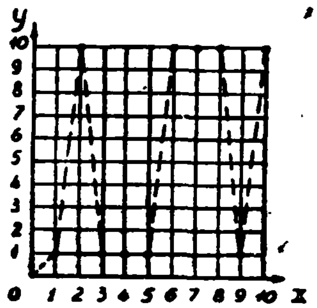

Fig. 1.

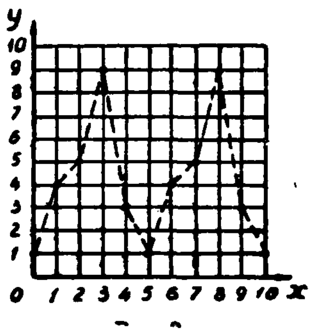

Fig. 2.

Page 23.

Theorem 3. If function $y=f(x), x\in Z, y\in Z$ is symmetrical relative to axis $x=a$ or center of symmetry (a, b), then corresponding to it function $y=F(|x|_p)=|f(x)|_p$ is symmetrical relative to the axis of symmetry $x=|a|_p$ or center of symmetries $A(|a|_p, |b|_p)$.

Proof. Since function y=f(x) has an axis of symmetry x=a, the f(a+x)=f(a-x). From the equality functions follows the equality their moduli/modules, i.e.

$$|f(a+x)|_p=|f(a-x)|_p=|f(p+a-x)|_p.$$

consequently,

$$F(|a+x|_p)=F(|p+a-x|_p)$$

or

$$F(||a|_p+|x|_p|_p)=F(||a|_p-|x|_p|_p).$$

that also proves the first assertion of theorem.

Let us assume that $y=f(x),\ x\in Z,\ y\in Z$ has the center o[illegible]mmetry (a, b), i.e.

$$b-f(a-x)=-b+f(a+x).$$

The equality ~~s~~ is not broken, if both parts of it are taken on modulus/module p, i.e.

$$|b-f(a-x)|_p=|-b+f(a+x)|_p$$

or.

$$|b-|f(a-x)|_p|_p=|-b+|f(a+x)|_p|_p.$$

Since

$$|f(x)|_p=F(|x|_p),$$

that

$$||b|_p-F(|a-x|_p)|_p=|-|b|_p+F(|a+x|_p)|_p$$

or

$$||b|_p+p-F(||a|_p-|x|_p+p|_p)|_p=|p-|b|_p+F(||a|_p+|x|_p|_p)|_p.$$

that also proves theorem.

Function $y=F(|x|_p)$ is called even, if

$$F(|x|_p)=F(|p-|x|_p|_p) \qquad (28)$$

for any $x\in Z$, i.e., it has an axis of symmetry x=0, and odd, if

$$|p-F(|p-|x|_p|_p)|_p=F(|x|_p) \qquad (29)$$

for any value $x \in Z$, i.e., it has a center of symmetry (0, 0).

Page 24.

The graph of even function $y=|2x^2|_{11}$ is shown on figure 3, odd $y=|x^3|_{11}$ - in figure 4.

Linear, exponential and index functions.

As examples let us consider the linear, exponential and index functions, which have important value in nomography.

Linear function on set $|Z|_p$ we will call the function of the form

$$|Ax+By+C|_p=0, \qquad (30)$$

where $A, B, C, x, y \in Z$.

Since

$$|Ax+By+C|_p=||A|_p|x|_p+|B|_p|y|_p+|C|_p|_p=0,$$

that values A, B, C, x, y can be examined from set $|Z|_p$. Expression (30) corresponds on plane $|\Pi_z|_p$ to the equation of the straight line whose properties were examined above.

The exponential function, determined on set $|Z|_p$, we will call the function of form.

$$y=|a^x|_p. \qquad (31)$$

Theorem 4. Function $y=|a^x|_p$, $x \in Z$ takes the different values with different $x \neq 0$, if a the -first-shaped root of number p.

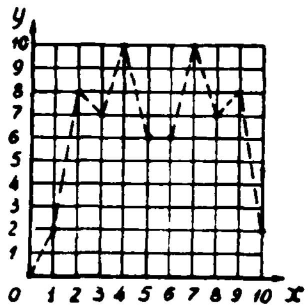

Fig. 3.

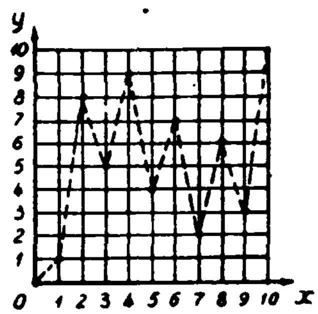

Fig. 4.

Page 25.

Proof. Since a is primitive roots, the comparison $a^x \equiv 1 \pmod p$ has unique solution. Then according to theorem 2.10 of work [4] comparison $a^x \equiv A \pmod p$, where A is not multiple p, also has unique solution.

Lemma 2. If α is the solution of comparison of $a^x \equiv 1 \pmod p$, where a is not primitive roots of p, then $p-1-\alpha$ is also the solution of this comparison.

Since $a^\alpha \cdot a^{-\alpha}=1$, then $a^\alpha \cdot a^{-\alpha} \equiv 1 (\mathrm{mod}\ p)$.

On the strength of the fact that

$$a^{-\alpha}=a^{p-1-\alpha} \tag{32}$$

expression (32) can be rewritten in the form.

$$a^{\alpha}\cdot a^{p-1-\alpha}\equiv 1(\mathrm{mod}\ p).$$

Since $a^{\alpha}\equiv 1(\mathrm{mod}\ p)$, then $a^{p-1-\alpha}\equiv 1(\mathrm{mod}\ p)$ and, therefore, p-1-α is the solution, QED.

theorem 5. Function $y=|a^x|_p,\ x\in Z$ is periodic, if a is not primitive roots of p.

Proof. Since a is not primitive roots of p, the comparison $a^x\equiv 1(\mathrm{mod}\ p)$ have, in view of lemma 2, at least two solutions:

$$x_1=p-1-\alpha \overset{(1)}{\text{и}} x_2=\alpha,\ \alpha<p-1.$$

Key: (1). and.

Let us demonstrate that p-1-α is the period of function $y=|a^x|_p$, i.e.,

$$|a^x|_p=|a^{x+p-1-\alpha}|_p.$$

But

$$|a^{p-1-\alpha}\cdot a^x|_p=||a^{p-1-\alpha}|_p\cdot|a^x|_p|_p=|a^x|_p,$$

since.

$$|a^{p-1-\alpha}|_p=1.$$

The example to exponential function $y=|6^x|_{11}$ is given in figure 5. As can be seen from graph (Fig. 5), the values of this function are equal only at points x=0 and x=10, moreover the values of function at these points are equal to one. Another example to exponential function $y=|4^x|_{11}$ is given in figure 2, where also at points x=0, x=10 and x=5 value of function they are equal to one. This function periodic, with the period l=5.

Index function we will call the function of the form

$$y=|\mathrm{ind}_a x|_p,\ a, x\in|Z|_p,$$

where a - primitive roots of number p, which is the basis/base of index.

Page 26.

Let us construct the graph (Fig. 6) of function $y=|\mathrm{ind}_a x|_p$ with a=6 and p=11, using the table of indices [4], which can be obtained by the solution of comparison of $6^x\equiv A \pmod{11}$. In comparing the graphs, given in figures 5 and 6, it is possible to note that the graph of index function is mirror reflection by relatively straight/direct $y=|x|_p$ of the graph of exponential function. Exception/elimination is only point (10, 1) of graph in figure 5, which is not in figure 6.

As is known, the graph of logarithmic function on plane Π is obtained from the graph of exponential function by mirror reflection by relatively straight/direct y=x. Consequently, index function can be interpreted as the logarithmic function, undertaken on modulus/module p, i.e., index function on set $|Z|_p$ corresponds to logarithmic function on set Z. Let us show that the index function on $|Z|_p$ possesses some properties of logarithmic function. For example, for the logarithmic function is correct the equality form $\log_a a=1$. The same equality is correct for the the index of function, i.e. $|\mathrm{ind}_a a|_p=1$

It is known that the plotted functions $y=\log_{\frac{1}{a}} x$ and $y=\log_a x$ are symmetrical relative to x axis. Let us construct plotted function $y=|\mathrm{ind}_{|\frac{1}{6}|_{11}} x|_{11}$. As can be seen from figures 6 and 7, functions $y=|\mathrm{ind}_{|\frac{1}{6}|_{11}} x|_{11}=|\mathrm{ind}_2 x|_{11}$ and $y=|\mathrm{ind}_6 x|_{11}$ are symmetrical relative to axis y=0. Consequently, plotted function $y=|\mathrm{ind}_{|\frac{1}{a}|_p} x|_p$ can be obtained from the plotted function $y=|\mathrm{ind}_a x|_p$ by its mirror reflection relative to axis y=0.

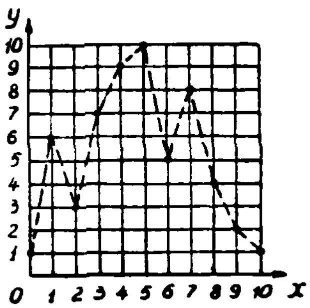

Fig. 5.

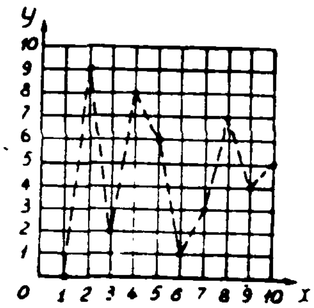

Fig. 6.

Page 27.

Lemma 3. Any prime/simple number p has an even quantity of primitive roots in cases when $a \neq \left|\frac{1}{a}\right|_p$, where a - primitive roots of number p.

Actually/really, since to different values x correspond the different values of function $y = |\mathrm{ind}_a x|_p$, where a - primitive roots of number p, and function $y = |\mathrm{ind}_{\left|\frac{1}{a}\right|_p} x|_p$ is symmetrical relative to axis y=0 of function $y = |\mathrm{ind}_a x|_p$, then to different values x correspond the different values of function $y = |\mathrm{ind}_{\left|\frac{1}{a}\right|_p} x|_p$ and the basis/base of index $\left|\frac{1}{a}\right|_p$ is also primitive roots of number p, that also proves lemma.

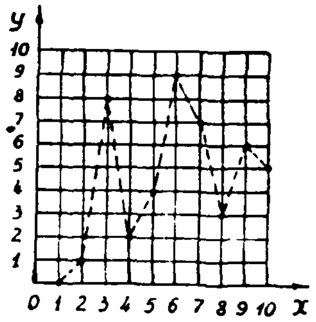

Fig. 7.

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Page 28.

Nomograms from the adjusted points in the deductions ~~on~~
modulo
~~modulus/module~~ p on plane $|\Pi_z|_p$

I. Ya. Akushskiy, A. Ya. E'yanzin.

In works [1, 2] were examined questions of the use/application of systems of nomograms in numeration system in the residual classes for the purpose of an increase in the accuracy of calculations. In this case each nomogram in modulus/module p differed from those existing in terms of the fact that to points with the integral marks were assigned the marks, equal to deduction on modulus/module p from the initial marks. Thus, during the construction of nomograms on the independent foundations on the answer scale appear different points with the identical marks, which introduces the unjustified redundancy into the representation of the answer scale and creates considerable difficulties during the use of nomograms for the reverse operations.

This work is dedicated to construction and research of nomograms from the adjusted points in the deductions on modulus/module p whose scales and resolving straight lines are constructed in the deductions according to modulus/module p. This makes it possible to reduce periodicity on the scales and gives the possibility to use nomograms for the reverse operations.

Scales.

Let us determine the scales on the plane of integers on modulus/module p $|\Pi_Z|_p$ analogously with the scales on euclidean plane Π. In work [3] are introduced the concepts of plane $|\Pi_Z|_p$, different functional dependences on set $|Z|_p$ and are traced their basic properties. The point of plane $|\Pi_Z|_p$ with the ascribed by it value of certain variable/alternating $u \in |Z|_p$, where $|Z|_p$ - set of the integers, undertaken on modulus/module p, let us name the marked point. The value of the variable/alternating u which is ascribed to given point, is called the mark of point.

Page 29.

Scale of variable/alternating $u \in |Z|_p$ - the locus of the marked points of plane $|\Pi_Z|_p$. Line on plane $|\Pi_Z|_p$, on which are placed the marked points, carrier of the scale. The scales, in which the equation of carrier is a linear function, we will consider linear, all others - nonlinear. The functional scale let us name the locus of the marked points whose location on the scale is determined by the character of the preset function.

The evenly divided scale.

In contrast to the evenly divided scales on plane Π, characterized by the fact that on them the lengths of segments are proportional to marks (for example, millimetric ruler, the scale of the division of circumference into the degrees), the evenly divided scales on plane $|\Pi_z|_p$ we will call the scales, for the coordinates of points of which are satisfied the condition

$$|y_{i+2}-y_{i+1}|_p=|y_{i+1}-y_i|_p,$$
$$x_{i+2}-x_{i+1}=x_{i+1}-x_i=1 \text{ для } \forall i\in|Z|_p.$$

Moreover it is assumed that $x_j>x_i$, if j>i for $\forall i,j\in|Z|_p$.

It is not difficult to note that any straight line, constructed on plane $|\Pi_z|_p$, will be the evenly divided scale, if we the mark of point consider the appropriate value of argument.

Construction of the functional scales.

The functional scales in the deductions on certain modulus/module p are constructed as follows. Let on set $|Z|_p$ be preset certain function $y=|f(x)|_p$. Let us compute all values of function and let us regulate them as follows:

$$f_0=|f(0)|_p$$
$$f_1=|f(1)|_p$$
$$\cdots\cdots$$
$$f_{p-1}=|f(p-1)|_p.$$

On plane $|\Pi_z|_p$ Let us consider certain straight line. For the fiducial mark A on the straight line let us take the point, which has coordinate x, equal to zero. Along the axis X we will plot/deposit segments with a length of $f_0, f_1, \ldots, f_{p-1}$ and assign to points with abscissas $f_0, f_1, \ldots, f_{p-1}$, that lie to the straight line (carrier of the scale), marks 0, 1, ..., p-1 respectively. Obtained thus scale will represent the functional scale in the deductions on modulus/module p.

Page 30.

Let us note some properties of the functional scales on planes $|\Pi_z|_p$, which differ significantly from the properties of the ordinary functional scales on plane Π.

1. Functional scale is discrete/digital and exists only at marked points, i.e., unmarked points of carrier of scale are not in any way connected with functional scale.

2. Marked points there can be not more than p, i.e., scale is limited.

3. Marks on carrier of scale can be placed nonmonotonically.

4. To some points of scale can be ascribed several marks.

The double scales.

If we combine two scales with the identical carrier one functional, second uniform, constructed on one and the same scale, then so that their fiducial marks would coincide, then we will obtain the double scales, i.e., the scale, which has two series/rows of marks. On this scale it is convenient to compute the values of functions for the different values of argument. For this it is necessary on the functional scale to find out the values of argument, and on the evenly divided scale at the same point to read the corresponding value of function. And vice versa, knowing the value of function, it is possible to determine the appropriate value of argument. For this it is necessary to find the mark, equal to the value of function on the evenly divided scale, and to read the confronting against it mark of the functional scale. The absence of mark against any value on the evenly divided scale tells about the fact that this value of function there does not exist in set $|Z|_p$ not for what value of argument from set $|Z|_p$.

Thus, the double scales is the simplest nomogram and serves for calculating the value of function. Its advantage before the tables of functions lies in the fact that the double scales is more demonstrative and it is conveniently used. The assignment of the double scales is equivalent to the assignment of table or plotted function.

Index scales.

The index scale we will call the functional scale, constructed on certain straight line on plane $|\Pi x|_p$ for function $y=|\mathrm{ind}_a x|_p$. This scale has the important value with the execution of arithmetic operations in the deductions on modulus/module p.

Page 31.

For an example let us construct the functional scale $y=|\mathrm{ind}_a x|_p$ with a=6 and p=11. The table of this function is given in work [4]. The index scale is shown on Figure 1.

The index scale is the set of points with the marks from 1 to p-1. The location of marks depends on the value of the basis/base of index (primitive roots) and on the value of modulus/module p. The first point of the scale has a mark, equal to unity, the second - mark a, i.e., the value of the basis/base of index, the p-1/2 point has a mark p-1. The latter/last, p-1 point of the scale has mark x, such, that $|a \cdot x|_p = 1$ when $x = \left|\frac{1}{a}\right|_p$. These points are characteristic for the index scale.

Nomograms from the adjusted points in the deductions on modulus/module p.

Nomogram from the adjusted points in the deductions on modulus/module p on plane $|\Pi_z|_p$ for the equation

$$|f(u, v, w)|_p = 0 \qquad (1)$$

let us name the nomogram whose every three values u, v, w, which satisfy initial equation (1), lie/rest on one straight line, carried out on plane $|\Pi_z|_p$, and vice versa, if u, v, w - mark of three points of nomogram, which lie on one straight line, then the set of three of numbers u, v, w satisfies equation (1). This straight line we will call the resolving straight line. The values of variable/alternating, entering equation (1), are represented on the nomogram as the points, which compose the scales of these variable/alternating. The form of nomogram, i.e., the form of its scales and their mutual location on plane $|\Pi_z|_p$, is determined by the form of the represented functional dependence and by the method of the construction of nomogram.

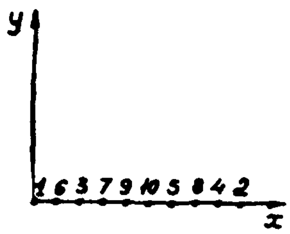

Fig. 1.

Page 32.

Theorem. If for the equation

$$f(u, v, w)=0 \qquad (2)$$

(where f(u, v, w) - full-valued function, u, v, w - integers) on the plane Π it is constructed the nomogram from the adjusted points, the equations of scales of which take in the following form:

$$\begin{aligned} u':&\quad x'_1=\varphi_1(u), \quad y'_1=\psi_1(u), \\ v':&\quad x_2'=\varphi_2(v), \quad y_2'=\psi_2(v), \qquad (3) \\ w':&\quad x_3'=\varphi_3(w), \quad y_3'=\psi_3(w), \end{aligned}$$

then for equation (1)

$$|f(u, v, w)|_p=0$$

nomogram on plane $|\Pi_z|_p$, giving the solution of equation (1), will have equations of the scales of following form:

$$\begin{aligned} u:&\ x_1=|\varphi_1(u)|_p, \quad y_1=|\psi_1(u)|_p, \\ v:&\ x_2=|\varphi_2(v)|_p, \quad y_2=|\psi_2(v)|_p, \qquad (4) \\ w:&\ x_3=|\varphi_3(w)|_p, \quad y_3=|\psi_3(w)|_p. \end{aligned}$$

so that

$$\left\| \begin{matrix} 1 & |\varphi_1(u)|_p & |\psi_1(u)|_p \\ 1 & |\varphi_2(v)|_p & |\psi_2(v)|_p \\ 1 & |\varphi_3(w)|_p & |\psi_3(w)|_p \end{matrix} \right\|_p = 0. \quad (5)$$

Proof. Since the nomogram, constructed for equation F(w, v, w)=0, is nomogram from expressed points, then, therefore, the functions, entering the equations of scale satisfy condition [5]

$$\begin{vmatrix} 1 & \varphi_1(u) & \psi_1(u) \\ 1 & \varphi_2(v) & \psi_2(v) \\ 1 & \varphi_3(w) & \psi_3(w) \end{vmatrix} = 0. \quad (6)$$

From the determination of implicit function in $|Z|_p$ [3] $|f(u, v, w)|_p = F(|u|_p, |v|_p, |w|_p)$ it follows that if u, v, w are the solutions of equation f(u, v, w)=0 that $|u|_p, |v|_p, |w|_p$ will be the solutions of equation $F(|u|_p, |v|_p, |w|_p)=0$, but regarding in $|Z|_p$ the function, preset in the parametric form, follows that for functions (3) corresponding expressions $|u'|_p, |v'|_p, |w'|_p$ will take form (4). Consequently, (4) it is the solution of equation (1).

Page 33.

Let us consider expression (6), undertaken on modulus/module p. Then in view of the properties of operations with the least positive remainders/residues we obtain:

$$\left\|\begin{matrix} 1 & \varphi_1(u) & \psi_1(u) \\ 1 & \varphi_2(v) & \psi_2(v) \\ 1 & \varphi_3(w) & \psi_3(w) \end{matrix}\right\|_p = \left\|\begin{matrix} 1 & |\varphi_1(u)|_p & |\psi_1(u)|_p \\ 1 & |\varphi_2(v)|_p & |\psi_2(v)|_p \\ 1 & |\varphi_3(w)|_p & |\psi_3(w)|_p \end{matrix}\right\|_p = 0. \qquad (7)$$

Substituting for $|\varphi_1(u)|_p$, $|\psi_1(u)|_p$, $|\varphi_2(v)|_p$, $|\psi_2(v)|_p$, $|\varphi_3(w)|_p$, $|\psi_3(w)|_p$ their value from (4), we will obtain:

$$\left\|\begin{matrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{matrix}\right\|_p = 0. \qquad (8)$$

This is the condition for the passage of straight line through three points (x_1, y_1), (x_2, y_2), (x_3, y_3) [3]. Consequently, the set of curves with equations (4) is nomogram from the adjusted points for equation (1).

Equation (7) they will satisfy all sets of three of numbers u, v, w, which are the marks of the points of nomogram, which lie on one straight line. But such sets of three of numbers compose all possible solutions of equation (1). Consequently, equation (5) contains among it solutions all solutions of equation (1).

Conversely, if it is possible to find the equation of form (5), which contains all solutions of equation (1), then for equation (1) on plane $|\Pi z|_p$ it is possible to construct the nomogram whose scales are determined by equations (4).

In such a way as for equation $|f(u, v, w)|_p = 0$ to construct nomogram from the adjusted points on plane $|\Pi z|_p$, is necessary to find this equation

$$\left\| \begin{vmatrix} 1 & |\varphi_1(u)|_p & |\psi_1(u)|_p \\ 1 & |\varphi_2(v)|_p & |\psi_2(v)|_p \\ 1 & |\varphi_3(w)|_p & |\psi_3(w)|_p \end{vmatrix} \right\|_p = 0, \qquad (5)$$

where on the left side it will be worthwhile the definition, undertaken on certain modulus/module p. Its first column consists of units, and each line contains the functions only of one variable/alternating which also are taken on modulus/module p. The values of the functions of the second column of determinant will give abscissas, while the value of the functions of the third column - the ordinate of the points of the scales of nomogram.

Page 34.

As it follows from the theorem and the properties of the scales on plane $|\Pi z|_p$, the solution of equation $|f(u, v, w)|_p = 0$ with the help of the nomograms from the adjusted points will be always exact, since the intersection with the resolving straight line with the answer scale occurs only at the marked point. Because of this during the solution of equations with the help of the nomograms, constructed on plane $|\Pi_z|_p$, it is not necessary to carry out interpolation for the determination of response/answer.

Classification of the nomographed equations.

Equation $|f(u, v, w)|_p=0$ we will call that nomographed on plane $|\Pi_z|_p$, if for it it is possible to find the equation of form (5).

The nomographic order of the nomographed equation is called a number of different functions of one variable/alternating in the equation, obtained from (5) after its development/scanning and simplification.

The lowest possible nomographic order - the third, since into equation with three variable/alternating must enter at least according to one function of each of the variable/alternating, highest - the sixth, when all functions in equation (5) different.

For example, the equations of the third nomographic order contain according to one function of each of three the variable/alternating u, v and w: $|f_1(u)|_p$, $|f_2(v)|_p$, $|f_3(w)|_p$, and they can be converted to certain simplest (canonical) form.

The equations of the form

$$|f_3(w)|_p = ||f_1(u)|_p \cdot |f_2(v)|_p|_p, \tag{9}$$

$$|f_3(w)|_p = ||f_1(u)|_p + |f_2(v)|_p|_p, \tag{10}$$

$$||f_1(u)|_p \cdot |f_2(v)|_p \cdot |f_3(w)|_p|_p = ||f_1(u)|_p + |f_2(v)|_p + |f_3(w)|_p|_p \tag{11}$$

we will call respectively the first, second and third canonical form of the equations of the third nomographic order.

Nomograms class also according to their genre. The genre of nomogram is called a number of nonlinear scales in it. The lowest possible genre of nomogram - zero, when all scales are linear. The highest possible genre - the third, when all three scales are nonlinear.

Page 35.

Nomograms of the first canonical form of the equations of the third nomographic order.

The first canonical form of the equations of the third nomographic order takes form (9):

$$|f_3(w)|_p = ||f_1(u)|_p \cdot |f_2(v)|_p|_p.$$

For it by three different methods it is possible to write the equations of form (5), which reduce to three forms of nomograms, essentially different in the geometric structure.

Nomograms of zero genre.

Let us register equation (5) for equation (9) in the following form:

$$\left\| \begin{matrix} 1 & 0 & |f_1(u)|_p \\ 1 & \left|\frac{1}{1-|f_2(v)|_p}\right|_p & 0 \\ 1 & 1 & |f_3(w)|_p \end{matrix} \right\|_p = 0. \qquad (12)$$

revealing the definition, which stands on the left side of equation (12), it is easy to check that equation (12) is equivalent to equation (9).

From equation (12) we obtain the simplest equations of the scales of nomogram on plane $|\Pi_z|_p$.

They take the form

$$\left\{ \begin{matrix} (1)\text{шкалы } u: & x_1=0 & y_1=|f_1(u)|_p \\ \text{шкалы } v: & x_2=\left|\frac{1}{1-|f_2(v)|_p}\right|_p & y_2=0 \\ \text{шкалы } w: & x_3=1 & y_3=|f_3(w)|_p \end{matrix} \right\}. \qquad (13)$$

Key: (1). the scale.

All three scales of nomogram are linear. Consequently, the nomogram of equation (10) with scales (13) is the nomogram of zero genre on plane $|\Pi_z|_p$. The scale of the variable/alternating u will be arranged/located on y axis, the variable/alternating v - on axis X and variable/alternating w - on the straight/direct, parallel y axis and distant behind it at a distance, equal to one.

As an example of the nomogram of the zero genre of the equation of the first canonical form let us give the nomogram of multiplication m•n of the natural series of numbers on modulus/module 11, depicted in figure 2. The points, which lie on the dotted line at the assemblies of reference grid, are points of the resolving straight line with multiplication $|7\times7|_{11}$.

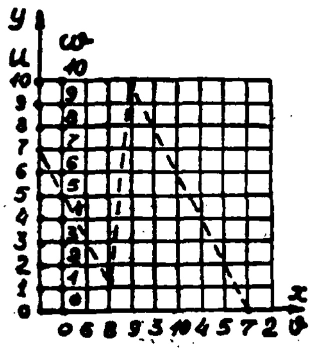

Fig. 2.

Page 36.

In this case the point of intersection of the resolving straight line and answer scale has a mark, equal to five.

Nomograms of the second genre.

Let us write equation (5) in the form

$$\left\| \begin{array}{ccc} 1 & |f_1(u)|_p & |f_1^2(u)|_p \\ 1 & |p-|f_2(v)|_p|_p & |f_2^2(v)|_p \\ 1 & 0 & |f_3(w)|_p \end{array} \right\|_p = 0. \qquad (14)$$

In view of the properties of operations with the smallest positive remainders/residues equation (14) equally to the equation

$$\left\|\begin{matrix} 1 & f_1(u) & f_1^2(u) \\ 1 & -f_2(v) & f_2^2(v) \\ 1 & 0 & f_3(w) \end{matrix}\right\|_p = 0. \qquad (15)$$

revealing the left side of equation (15), we will obtain

$$||f_2(v)+f_1(u)|_p \cdot |f_1(u)\cdot f_2(v)-f_3(w)|_p|_p=0. \qquad (16)$$

Whence it follows that the solutions of equation (9) are the solutions of equation (16), and also, therefore, equation (14). Consequently, equation (5) for equation (9) can be registered in the form (12).

The equations of the scales of nomogram will be the following:

$$\left.\begin{matrix} {}^{(1)}\text{шкалы } u\text{:} & x_1=|f_1(u)|_p, & y_1=|f_1^2(u)|_p \\ {}^{(1)}\text{шкалы } v\text{:} & x_2=|p-|f_2(v)|_p|_p, & y_2=|f_2^2(v)|_p \\ {}^{(1)}\text{шкалы } w\text{:} & x_3=0, & y_3=|f_3(w)|_p \end{matrix}\right\}. \qquad (17)$$

Key: (1). the scale.

Excluding the variable/alternating u from the equations of scale u, let us find the equation of the carrier of scale u:

$$y=|x^2|_p.$$

Exception/elimination of the variable/alternating v from the equations of scale v gives the same result. Thus, the scales of the variable/alternating u and v have general/common/total carrier. The carrier of scale w is the straight line, which coincides with the axis of ordinates.

As can be seen from the equations of the scales, this is the nomogram of the second genre, since the carriers of scales u and v are nonlinear.

Page 37.

As an example of the nomogram of the second equation of the first canonical form let us give the nomogram of multiplication m•n of the natural series of numbers cn modulus/module 11, depicted in figure 3. since scales u and v coincide, let us agree upper mark to carry to scale u, and lower - to scale v. Dotted line showed the resolving straight line with multiplication $|7\times7|_{11}=5.$ Analogously it is possible to consider and to construct nomograms for all remaining canonical forms.

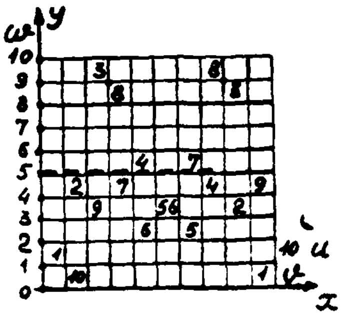

Fig. 3.

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Page 38.

Mathematical simulation of the thermal work of soaking pit.

B. R. Amangel'diyev.

In works [1, 2] is proposed the mathematical model of the process of heating metal in the soaking pit. For describing the unsteady temperature fields in the laying and the heated metal are used the equations of thermal conductivity with the boundary conditions, the reflecting results of the contemporary theory of radiation heat exchange [3, 4]. As the basis of the construction of model is assumed zonal method [5, 6], in this case with a number of surface zones are connected the lateral faces of ingots and the surface of laying, and as the volumetric zone is selected the internal space of well, which contains the heating environment.

Page 39.

The mathematical formulation of task takes the form:

$$c_1(q_1)\rho_1(q_1)\frac{\partial q_1(x_1, t)}{\partial t} = \frac{\partial}{\partial x_1}\left(\lambda_1(q_1)\frac{\partial q_1(x_1, t)}{\partial x_1}\right), \tag{1}$$

$$0 < x_1 < l_1, \ t > 0,$$

$$q_1(x_1, 0) = q_1^0(x_1), \tag{2}$$

$$-\lambda_1(q_1)\frac{\partial q_1(0, t)}{\partial x_1} = \sigma_0 A_1 A_2 \psi_{11}[q_2^4(l_2, t) - q_1^4(0, t)] + \tag{3}$$

$$+4\sigma_0 A_1 \alpha \psi_{12}[u^4(t) - q_1^4(0, t)],$$

$$\lambda_1(q_1)\frac{\partial q_1(l_1, t)}{\partial x_1} = \sigma_0 A_1 A_2 \psi_{11}[q_2^4(l_2, t) - q_1^4(l_1, t)] + \tag{4}$$

$$+4\sigma_0 A_1 \alpha \psi_{12}[u^4(t) - q_1^4(l_1, t)],$$

$$\frac{\partial q_2(x_2, t)}{\partial t} = a_2 \frac{\partial^2 q_2(x_2, t)}{\partial x_2^2}, \ 0 < x_2 < l_2, \tag{5}$$

$$q_2(x_2, 0) = q_2^0(x_2), \tag{6}$$

$$-\lambda_2 \frac{\partial q_2(0, t)}{\partial x_2} = \alpha_{20}[u_н - q_2(0, t)], \tag{7}$$

$$\lambda_2 \frac{\partial q_2(l_2, t)}{\partial x_2} = 4n\sigma_0 A_2 A_1 \psi_{21}[q_1^4(l_1, t) - q_2^4(l_2, t)] + \tag{8}$$

$$+4\sigma_0 A_2 \alpha \psi_{22}[u^4(t) - q_2^4(l_2, t)],$$

$$u(t) = [\eta Q v(t) + \rho c q_г v(t) + \rho c \gamma q_н v(t) + 4\sigma_0 A_1 \alpha \psi_{12}(q_1(l_1, t) +$$

$$+u(t))(q_1^2(l_1, t) + u^2(t))F_1 n q_1(l_1, t) + 4\sigma_0 A_2 \alpha \psi_{22}(q_2(l_2, t) +$$

$$+u(t))(q_2^2(l_2, t) + u^2(t))F_2 q_2(l_2, t)] \cdot [(1+\gamma)\rho c v(t) + \tag{9}$$

$$+4\sigma_0 A_1 \alpha \psi_{12}(q_1(l_1, t) + u(t))(q_1^2(l_1, t) + u^2(t))F_1 n +$$

$$+4\sigma_0 A_2 \alpha \psi_{22}(q_2(l_2, t) + u(t))(q_2^2(l_2, t) + u^2(t))F_2]^{-1},$$

where $q_1, q_2, u, u_н$ - temperature of ingot, laying, heating and external environment respectively; l_1, l_2 - size/dimension of the cross section of ingot and the thickness of laying; c_1, λ_1, λ_2, α_2 - thermophysical coefficients; ρ_1 - density of metal; F_1 F_2 - surface area of ingot and laying; $\psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}$ - generalized coefficients of radiation heat exchange; α - degree of the absorption of air-gas environment; γ - coefficient of the flow rate of air (it is accepted equal to the product of the real coefficient of expenditure for a quantity of air, theoretically necessary for combusting the unit volume of fuel/propellant [7]); ρ, c - density and the heat capacity of air-gas environment; v(t) - the consumption of gas fuel; Q - fuel heating value; q_r and q_a - temperature of gas and air at the input into the well; n - quantity of ingots in the well; σ_0 - constant of Stephan - Boltzmann; A_1 and A_2 - degree of the dark of the surfaces of ingot and load; x_1 and x_2 - space coordinates; t - time.

It is necessary to determine the temperature states of ingots and laying under the preset law of the admission of fuel/propellant v=v(t).

This article examines a question, that concern computational aspects and specific applications of stated problem.

Page 40.

Computing circuit of the solution of problem.

We convert the boundary conditions (3), (4) and (8) to the form

$$-\lambda_1(q_1)\frac{\partial q_1(0,t)}{\partial x_1}=\alpha_{11}[q_2(l_2,t)-q_1(0,t)]+\alpha_{12}[u(t)-q_1(0,t)], \quad (10)$$

$$\lambda_1(q_1)\frac{\partial q_1(l_1,t)}{\partial x_1}=\alpha_{11}[q_2(l_2,t)-q_1(l_1,t)]+\alpha_{12}[u(t)-q_1(l_1,t)], \quad (11)$$

$$\lambda_2\frac{\partial q_2(l_2,t)}{\partial x_2}=\alpha_{21}[q_1(l_1,t)-q_2(l_2,t)]+\alpha_{22}[u(t)-q_2(l_2,t)], \quad (12)$$

where

$$\alpha_{11}=\sigma_0 A_1 A_2 \psi_{11}[q_1(0,\ t)+q_2(l_2,\ t)][q^2_1(0,\ t)+q_2^2(l_2,\ t)],$$
$$\alpha_{12}=4\sigma_0 A_1 \alpha\psi_{12}[q_1(0,\ t)+u(t)][q_1^2(0,\ t)+u^2(t)], \quad (13)$$
$$\alpha_{21}=4n\sigma_0 A_2 A_1 \psi_{21}[q_2(l_2,\ t)+q_1(l_1,\ t)][q_2^2(l_2,\ t)+q_1^2(l_1,\ t)],$$
$$\alpha_{22}=4\sigma_0 A_2 \alpha\psi_{22}[q_2(l_2,\ t)+u(t)][q_2^2(l_2,\ t)+u^2(t)].$$

Then instead of expression (9) we obtain

$$u(t)=[\gamma Qv(t)+\rho cq_r v(t)+\rho c\gamma q_s v(t)+\alpha_{12}F_1 nq_1(l_1,\ t)+ \\ +\alpha_{22}F_2 q_2(l_2,\ t)][(1+\gamma)\rho cv(t)+\alpha_{12}F_1 n+\alpha_{22}F_2]^{-1}. \quad (14)$$

Using, according to [8], the replacement of the variable/alternating

$$\theta=\int_0^{q_1}\lambda_1(\varsigma)d\varsigma, \quad (15)$$

from equation (1) we will obtain

$$\frac{\partial\theta(x_1,t)}{\partial t}=a_1(q_1)\frac{\partial^2\theta(x_1,t)}{\partial x_1^2}, \quad (16)$$

where a_1 - coefficient of the thermal diffusivity of the heated metal.

Boundary conditions (10) and 11) respectively take the form

$$-\frac{\partial\theta(0,t)}{\partial x_1}=\alpha_{11}[q_2(l_2,\ t)-q_1(0,\ t)]+\alpha_{12}[u(t)-q_1(0,\ t)], \quad (17)$$

$$\frac{\partial\theta(l_1,t)}{\partial x_1}=\alpha_{11}[q_2(l_2,\ t)-q_1(l_1,\ t)]+\alpha_{12}[u(t)-q_1(l_1,\ t)]. \quad (18)$$

Page 41.

After using to equation (16) implicit grid diagram [8, 9], we will obtain

$$\frac{\theta_i^{j+1}-\theta_i^j}{\tau}=a_1(q_{1,i}^j)\ \frac{\theta_{i-1}^{j+1}-2\theta_i^{j+1}+\theta_{i+1}^{j+1}}{h_1^2},\ i=1,\ldots,m-1,$$

where h_1 - step/pitch along the axis x_1; τ - step/pitch on the time;

$$\theta_i^j=\theta(ih_1,\ j\tau).$$

Further conversions of equation lead to the expression

$$A_{1,i}\theta_{i-1}^{j+1}-C_{1,i}\theta_i^{j+1}+B_{1,i}\theta_{i+1}^{j+1}=-F_{1,i},$$

where

$$\left.\begin{aligned}&A_{1,i}=B_{1,i}=\frac{a_1(q_{1,i}^j)\tau}{h_1^2},\quad C_{1,i}=1+2\frac{a_1(q_{1,i}^j)\tau}{h_1^2},\\&F_{1,i}=\theta_i^j,\quad i=1,\ldots,m-1.\end{aligned}\right\}\qquad(19)$$

For solving the obtained system of algebraic equations we use a dispersion method [8]. Thus, we have

$$\theta_i^{j+1}=\alpha_{1,\,i+1}\theta_{i+1}^{j+1}+\beta_{1,\,i+1},\quad i=m-1,\ldots,1,0,\qquad(20)$$

where

$$\alpha_{1,\,i+1}=\frac{B_{1,i}}{C_{1,i}-A_{1,i}\alpha_{1,i}},\quad \beta_{1,\,i+1}=\frac{A_{1,i}\beta_{1,i}+F_{1,i}}{C_{1,i}-A_{1,i}\alpha_{1,i}},\qquad(21)$$
$$i=1,\ldots,m-1.$$

The finite-difference approximation of boundary conditions let us lead with the help of method [9], which consists of the following.

We have

$$\theta(h_1,\ t)=\theta(0,\ t)+\frac{\partial\theta(0,\ t)}{\partial x_1}h_1+\frac{1}{2}\ \frac{\partial^2\theta(0,\ t)}{\partial x_1^2}\ h_1^2+O(h_1^3)$$

or

$$\frac{\theta_1-\theta_0}{h_1}=\frac{\partial\theta(0,\ t)}{\partial x_1}+\frac{1}{2}\ \frac{\partial^2\theta(0,\ t)}{\partial x_1^2}h_1+O(h_1^2).$$

Page 42.

After replacing the second derivative of θ on x_1 in the right side of the obtained relationship/ratio with the left side of equation (16), we will obtain

$$\frac{\theta_1-\theta_0}{h_1}-\frac{1}{2}\frac{h_1}{a_1(q_{1,0})}\frac{\partial\theta(0,t)}{\partial t}=\frac{\partial\theta(0,t)}{\partial x_1}+O(h_1^2).$$

Let us substitute the left side of this expression under boundary condition (17):

$$-\left(\frac{\theta_1^{j+1}-\theta_0^{j+1}}{h_1}-\frac{1}{2}\frac{h_1}{a_1(q^j_{1,0})}\frac{\theta_0^{j+1}-\theta_0^j}{\tau}\right)=\alpha_{11}(q^j_{2,r}-$$

$$-q^j_{1,0})+\alpha_{12}(u^j-q^j_{1,0}).$$

Let us note that $q_{1,0}$ and $q_{1,m}$ - temperature on the surfaces of ingot; $q_{2,0}$ and $q_{2,r}$ - temperature respectively on the external and internal surfaces of laying.

From the latter/last relationship/ratio we have

$$\theta_0^{j+1}=\frac{1}{1+\frac{1}{2}\frac{h_1^2}{a_1(q^j_{1,0})\tau}}\theta_1^{j+1}+\frac{\frac{1}{2}\frac{h_1^2}{a_1(q^j_{1,0})\tau}\theta_0^j+\alpha_{11}h_1(q^j_{2,r}-}{1+}$$

$$\frac{-q^j_{1,0})+\alpha_{12}h_1(u^j-q^j_{1,0})}{+\frac{1}{2}\frac{h_1^2}{a_1(q^j_{1,0})\tau}}.$$

after comparing this expression with (20) at the appropriate value of index i, we will obtain

$$\alpha_{1,1}=\frac{1}{1+\frac{1}{2}\frac{h_1^2}{a_1(q^j_{1,0})\tau}},$$

$$\beta_{1,1}=\frac{\frac{1}{2}\frac{h_1^2}{a_1(q^j_{1,0})\tau}\theta_0^j+a_{11}h_1(q^j_{2,r}-q^j_{1,0})+a_{12}h_1(u^j-q^j_{1,0})}{1+\frac{1}{2}\frac{h_1^2}{a_1(q^j_{1,0})\tau}}. \quad (22)$$

For right boundary condition (18) we have

$$\theta(l_1-h_1,\ t)=\theta(l_1,\ t)+\frac{\partial\theta(l_1,\ t)}{\partial x_1}(-h_1)+\frac{1}{2}\frac{\partial^2\theta(l_1,\ t)}{\partial x_1^2}(-h_1)^2+O(h_1^3)$$

or

$$\frac{\theta_m-\theta_{m-1}}{h_1}+\frac{1}{2}\frac{h_1}{a_1(q_{1,m})}\frac{\partial\theta(l_1,\ t)}{\partial t}=\frac{\partial\theta(l_1,\ t)}{\partial x_1}+O(h_1^2).$$

Page 43.

Then boundary condition (18) can be represented in the form

$$\frac{\theta_m^{j+1}-\theta_{m-1}^{j+1}}{h_1}+\frac{1}{2}\frac{h_1}{a_1(q^j_{1,m})}\frac{\theta_m^{j+1}-\theta^j_m}{\tau}=\alpha_{11}(q^j_{2,r}-$$

$$-q^j_{1,m})+\alpha_{12}(u^j-q^j_{1,m}).$$

From the latter/last expression, using (20), we will obtain

$$\theta_m^{j+1}=\frac{\frac{1}{2}\frac{h_1^2}{a_1(q^j_{1,m})\tau}\theta^j_m+\beta_{1,m}+\alpha_{11}h_1(q^j_{2,r}-q^j_{1,m})+}{1+\frac{1}{2}\frac{h_1^2}{a_1(q^j_{1,m})\tau}-}$$

$$\frac{+\alpha_{12}h_1(u^j-q^j_{1,m})}{-\alpha_{1,m}}. \qquad (23)$$

For the transition from θ to q_1 on the basis of expression (15) it is possible to register

$$\theta_i=\lambda_1(\xi_i q_{1,i})q_{1,i},\ i=0,1,\ldots,m,$$

where parameters $\xi_i \in [0, 1]$ can be determined with the help of the procedures of the search for root.

However, by us for simplicity of calculations is used the expression of the form

$$\theta_i^{j+1}=\bar{\lambda}^j_{1,i}\,q^{j+1}_{1,i},$$

where $\bar{\lambda}^j_{1,i}$ - average/mean value λ_1 in the segment from 0 to $q_1(ih_1, j\tau)$.

Thus, we have

$$q^{j+1}_{1,i}=\frac{1}{\bar{\lambda}^j_{1,i}}\theta_i^{j+1},\quad i=0,1,\ldots,m. \qquad (24)$$

For solving equation (5) is used the implicit grid diagram:

$$\frac{q_{2,k}^{j+1}-q_{2,k}^{j}}{\tau}=a_2\cdot\frac{q_{2,k-1}^{j+1}-2q_{2,k}^{j+1}+q_{2,k+1}^{j+1}}{h_2^2},\quad k=1,\ldots,r-1,$$

where h_2 - step/pitch along the axis x_2: $q^j_{2,k}=q_2(kh_2, j\tau)$.

Page 44.

Let us present it in the standard form:

$$A_{2,k}\,q_{2,k-1}^{j+1}-C_{2,k}q_{2,k}^{j+1}+B_{2,k}q_{2,k+1}^{j+1}=-F_{2,k},$$

where

$$A_{2,k}=B_{2,k}=\frac{a_2\tau}{h_2^2},\quad C_{2,k}=1+2\frac{a_2\tau}{h_2^2},\quad F_{2,k}=q^j_{2,k}. \qquad (25)$$

The solution of the obtained system of algebraic equations is realized also with the help of the dispersion method. We have

$$q_{2,k}^{j+1}=\alpha_{2,k+1}q_{2,k+1}^{j+1}+\beta_{2,k+1},\quad k=r-1,\ldots,1,0, \qquad (26)$$

where

$$\alpha_{2,k+1}=\frac{B_{2,k}}{C_{2,k}-A_{2,k}\alpha_{2,k}},\quad \beta_{2,k+1}=\frac{A_{2,k}\beta_{2,k}+F_{2,k}}{C_{2,k}-A_{2,k}\alpha_{2,k}}. \qquad (27)$$

For the finite-difference approximation of the boundary ones of arbit. equation (5) let us use the method, described above. Boundary condition (7) is represented in the form

$$-\lambda_2\left(\frac{q_{2,1}^{j+1}-q_{2,0}^{j+1}}{h_2}-\frac{1}{2}\,\frac{h_2}{a_2}\,\frac{q_{2,0}^{j+1}-q_{2,0}^{j}}{\tau}\right)=\alpha_{20}(u_{\text{н}}-q_{2,0}^{j+1}).$$

Hence we have

$$q_{2,0}^{j+1}=\frac{\lambda_2}{\lambda_2+\frac{1}{2}\frac{\lambda_2 h_2^2}{a_2\tau}+a_{20}h_2}q_{2,1}^{j+1}+\frac{\frac{1}{2}\frac{\lambda_2 h_2^2}{a_2\tau}q^j{}_{2,0}+a_{20}h_2u_\text{н}}{\lambda_2+\frac{1}{2}\frac{\lambda_2 h_2^2}{a_2\tau}+a_{20}h_2}.$$

after comparing this expression with (26) at the necessary value of index k, we will obtain

$$\alpha_{2,1}=\frac{\lambda_2}{\lambda_2+\frac{1}{2}\frac{\lambda_2 h_2^2}{a_2\tau}+a_{20}h_2},\quad \beta_{2,1}=\frac{\frac{1}{2}\frac{\lambda_2 h_2^2}{a_2\tau}q^j{}_{2,0}+a_{20}h_2u_\text{н}}{\lambda_2+\frac{1}{2}\frac{\lambda_2 h_2^2}{a_2\tau}+a_{20}h_2}. \qquad (28)$$

Boundary condition (12) after the necessary conversions is represented in the form

$$\lambda_2\left(\frac{q_{2,r}^{j+1}-q_{2,r-1}^{j+1}}{h_2}+\frac{1}{2}\frac{h_2}{a_2}\frac{q_{2,r}^{j+1}-q_{2,r}^{j}}{\tau}\right)=a_{21}(q_{1,m}^{j+1}-$$
$$-q_{2,r}^{j+1})+a_{22}(u^j-q_{2,r}^{j+1}).$$

From this expression and equation (26) we will obtain

$$q_{2,r}^{j+1}=\frac{\frac{1}{2}\frac{\lambda_2 h_2^2}{a_2\tau}q^j{}_{2,r}+\lambda_2\beta_{2,r}+a_{21}h_2q_{1,m}^{j+1}+a_{22}h_2u^j}{\lambda_2+\frac{1}{2}\frac{\lambda_2 h_2^2}{a_2\tau}+a_{21}h_2+a_{22}h_2-\lambda_2\alpha_{2,r}}. \qquad (29)$$

Page 45.

As a result the system of equations (19)-(29) together with initial conditions (2), (6) and relationships/ratios (13), (14) gives to us the mathematical model of the process of heating metal in the soaking pit, represented in the discrete/digital form.

In the implementation the tasks on TsVM of the dependence of the thermophysical coefficients a_1 and λ_1 on the temperature are represented by the piecewise-linear expressions

$$\lambda_1(q_{1,i})=\lambda_k+\frac{\lambda_{k+1}-\lambda_k}{q_{k+1}-q_k}(q_{1,i}-q_k), \qquad (30)$$

$$a_1(q_{1,i})=a_k+\frac{a_{k+1}-a_k}{q_{k+1}-q_k}(q_{1,i}-q_k),$$

if

$$q_k<q_{1,i}\leqslant q_{k+1},$$

$$\lambda_1(q_{1,i})=\lambda_0, \quad a_1(q_{1,i})=a_0, \qquad (31)$$

if

$$q_{1,i}\leqslant q_0;$$

$$\lambda_1(q_{1,i})=\lambda_N, \quad a_1(q_{1,i})=a_N, \qquad (32)$$

if

$$q_{1,i}>q_N,$$

where k=0, 1, ..., N; λ_k, a_k, q_k the numerical sequences, obtained on the basis of the preset dependences $a_1=a_1(q_1)$ and $\lambda_1=\lambda_1(q_1)$.

The calculation of the coefficients of radiation heat exchange $\psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}$ is connected with the optical and geometric structure of heating device/equipment. The geometry of object is determined by form and sizes/dimensions of well, by form, quantity and sizes/dimensions of the heated ingots and by character of their arrangement/position in well.

Coefficients $\psi_{11}, \ldots, \psi_{22}$ are calculated by two methods: 1) by the formulas, given in works [3, 4], 2) with the help of the method for statistical testing [10].

In resolving the equations of the dynamics of the mathematical model of object appears the need for determining the temperature of gaseous medium from relationship/ratio (14) under the conditions (13).

Page 46.

For this purpose the subtask indicated is reduced to the search for the root of the expression

$$y_2(u)=y_1(u)-y_3(u), \tag{33}$$

where

$$y_1(u)=\frac{d_1+d_2\tau_{12}(u)+d_3\tau_{22}(u)}{b_1+b_2\tau_{12}(u)+b_3\tau_{22}(u)} \tag{34}$$

- the right side of equation (14), and $y_3(u)=u$ - left.

Let us demonstrate the existence of root and its uniqueness. It is easy to establish/install the following: 1) with $u=0$, $y_2>0$, since $y_1(0)>0$, $y_3(0)=0$; 2) with $u\to\infty$, $y_1\to$const, a $y_3\to\infty$. Consequently, will be located such values u, at which $y_2<0$.

Hence immediately ensues/escapes/flows out the existence of the root of expression (33).

Since for of the real values of correlation coefficients (34) take the place of the condition

$$d_i>b_i \quad (i=1, 2, 3),$$

then y_1 is the monotonically increasing function from u. From this fact, and also point/item of 2 present proofs follows the uniqueness of root.

A procedure of the search for the root of equation (33) have realized we with the help of the method of "golden section" [11].

Similarly, one step/pitch of the solution of the problem in question is represented in the form of the following set of the operations (we assume that all necessary values toward the moment/torque of executing j+1 the step/pitch are known).

1. From relationships/ratios (13) and (14) with the help of procedure for finding root we determine temperature of gaseous medium u(t) and coefficients $\alpha_{11}, \ldots, \alpha_{22}$.

2. We find $a_1(q_1)$ and $\lambda_1(q_1)$. We compute on (19) $A_{1,i}$, $B_{1,i}$, $C_{1,i}$, $F_{1,i}$ $(i=1, \ldots, m-1)$.

3. We count from expressions (22) $\alpha_{1,1}$ and $\beta_{1,1}$. Then on (21) we calculate the value of auxiliary coefficients $\alpha_{1,i}$ and $\beta_{1,i}$ in the assemblies of grid region from left to right.

4. According to formula (23) we compute θ_m^{j+1}. With the help of dispersion relationship/ratio (20) we determine θ_i on the assemblies of grid region from right to left. Using expression (24), we obtain the temperature distribution in the ingot at j+1 step/pitch on the time.

5. We find values of coefficients $\alpha_{2,1}$ and $\beta_{2,1}$ on the basis (28). In formulas (27) we find coefficients $\alpha_{2,k}$ and $\beta_{2,k}$ from left to right, after determining preliminarily from relationships/ratios (25) $A_{2,k}$, $B_{2,k}$, $C_{2,k}$, $F_{2,k}$.

6. On formula (29) we compute temperature on internal surface of laying $q_{2,r}^{j+1}$. Using dispersion relationship/ratio (26), we determine the temperature in the laying on the assemblies of grid region from right to left.

Page 47.

Algorithm of standard mode/conditions.

In the practice of metallurgical production are applied the diverse modes/conditions of heating metal in soaking pits [12].

We have reproduced and by the help of the worked out model the following mode/conditions of heating: maximum heat output in the first period and prescribed constant temperature on the surface of ingot in the second period.

Mathematical model (19)-(29), (2), (6), (13), (14) in the program, comprised in an alpha-language, is designed as procedure H. The algorithm, which realizes the selected mode/conditions of heating, is represented in the form of a block-program in Figure 1.

Let us clarify the work of algorithm. In the process of heating is realized checking the condition

$$q_1(l_1 t)>D, \qquad (35)$$

where D - preset temperature of the surface of ingot. If this condition on is satisfied, then occurs transition into the beginning of program to procedure H.

But if this condition is satisfied, then is conducted the search for such feed of fuel/propellant, during which condition (35) will be broken. The named search is designed as the search for the root of the expression

$$y=D-\varepsilon_1-q_1(l_1,\ t), \qquad (36)$$

where $\varepsilon_1>0$ - certain assigned number.

It is obvious that with satisfaction of condition (35) $y<0$, and with

$$q_1(l_1,\ t)+\varepsilon_1<D \qquad (37)$$

$y>0$.

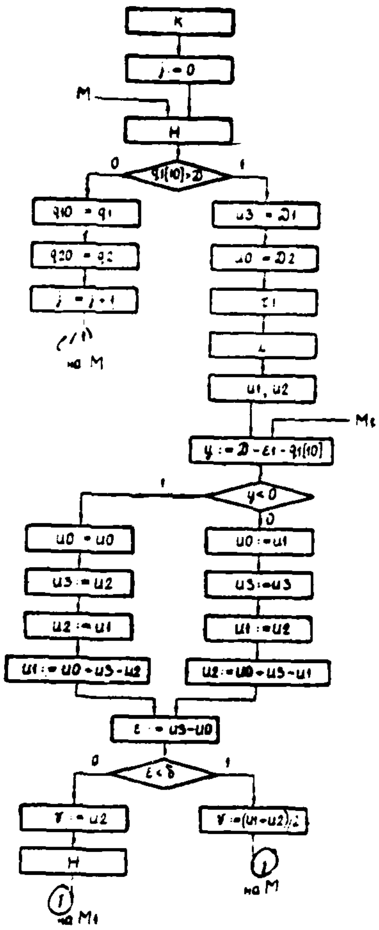

Fig. 1.

Key: (1). on.

Page 48.

For the solution of the problem indicated is used the method of "golden section" [11].

In the process of the work of program under condition (35) is conducted checking the relationship/ratio

$$\varepsilon<\delta, \quad (38)$$

where ε - value of the interval of uncertainty/indeterminancy, δ - preset accuracy of the search for root.

If $\varepsilon\geqslant\delta$, then the search for root is continued for transition to label M_1, otherwise we have (Fig. 1)

$$v=\frac{1}{2}(u_1+u_2) \quad (39)$$

and we pass to label M for the access to procedure H.

Results of numerical solutions.

Calculations are carried out for the well by the size/dimension 5.2x4.5x3.15 of m and ingots they became the brand of 08 KP the size/dimension 1.2x0.7x1.86 of m of the type VS 13.3. The coefficient of the volume absorption of the heating environment α is accepted equal to 0.11 1/m, and emissivity factor of the surfaces of ingots and laying - equal to 1.

Are preliminarily calculated the coefficients of radiation heat exchange. For the case of heating eight ingots they have the following values:

$$\psi_{11}=0{,}534;\ \psi_{12}=0{,}350;\ \psi_{21}=0{,}008;\ \psi_{22}=0{,}824.$$

The functional dependence of the thermophysical characteristics of steel of the brand of 08 KP on the temperature is constructed on the basis of data of work [13].

Are selected the following values of the thermophysical parameters of the material of laying $\lambda_2=1.31$ W/m•deg, $a_2=0.0023$ m²/h [14].

Calculations are carried out at the values of the parameters of the model: 1) Q=1856 W/m³•h, 2) $q_r=600°K$; 3) $\bar{q}_s=1000°K$; 4) $\gamma=1.61$ (taking into account the observation, given above); 5) $\rho c=0.40$ W•h/m³•deg, 6) $\eta=0{,}8$; 7) v=4000 m³/h (at the maximum heat output of well); 8) $F_1=7.07$ m²; 9) $F_2=108$ m²; 10) $l_1=0.9$ m; 11) $l_2=0.7$ m; 12) $u_s=293°K$; 13) D=1523°K; 14) $\alpha_{18}=11{,}63$ W/m²•deg.

Calculations are carried out on TsVM BESM-3M. Examples of the solutions (heating eight ingots of hot settlement) are given in figures 2 and 3, where 1 - consumption of gas fuel, 2 - temperature of gaseous medium in the well, 3 - temperature of the internal surface of laying, 4 - temperature on the surface of ingot, 5 - temperature on the axis of ingot.

Page 49.

The figure 2 corresponding to the high enthalpy of laying into the the initial moment in time, and figure 3 - low.

The analysis of results gives grounds to do following conclusions.

1. Worked out algorithm of simulation of standard mode/conditions correctly realizes one of used in practice modes/conditions of heating.

2. Common picture of course of process of heating metal in soaking pit, according to data of calculations, will be coordinated well with physical representations about object of study.

3. In all cases is very clearly visible period of languor.

4. Initial enthalpy of laying, other conditions being equal, determines, actually, entire course of the process of heating metal, especially sharply having effect on heating time.

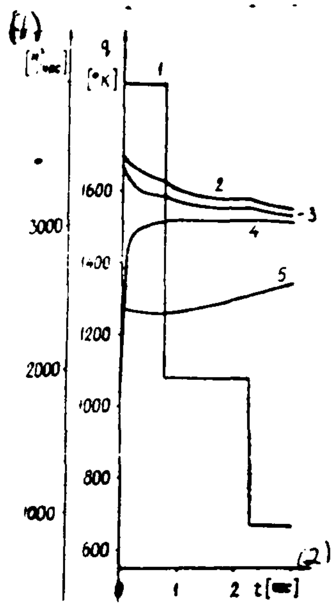

Fig. 2.

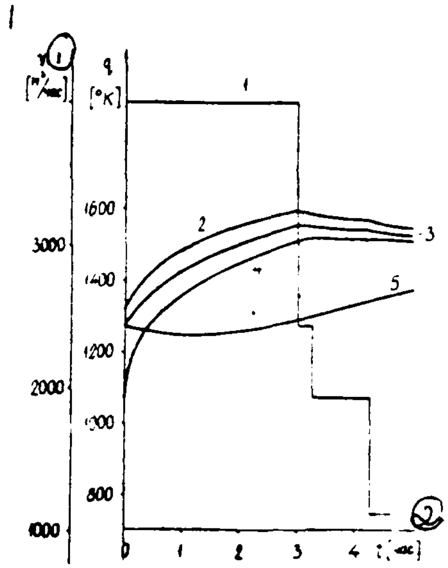

Fig. 3.

Key: (1). m/h. (2). hour.

Page 50.

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Page 51.

Interpolation codes, which correct errors.

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Finite-difference diagram and task of self correction.

Long before the emergence of the theory of the codes, which correct errors, in the theory of interpolation were used the methods of detection and correction of erroneous values in the tables of functions. The capability of interpolation procedures for the correction of errors most brightly is exhibited during the equidistant selection of the assemblies of interpolation. This fact is discovered due to the specific behavior of finite differences in the higher orders. Are traced below some versions of the construction of the codes, which correct errors, in light of the capability of interpolation procedures indicated for self correction. Let us begin from an illustrative example.

Example. Let us consider the field of deductions on modulus/module p=11. Let be preset the sequence of symbols of nine elements of the full/total/complete system of deductions on this modulus/module (3, 7, 9, 4, 5, 1, 8, 3, 2). Let us compose for this sequence the table of finite differences.

Let us further continue the table of differences, after filling in it the vacant places.

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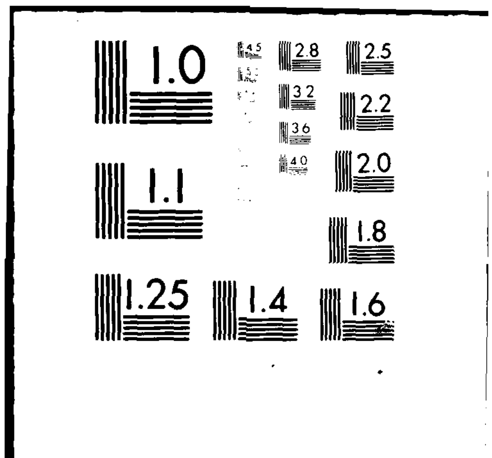
2 of 4



MICROCOPY RESOLUTION TEST CHART

f	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6	Δ^7	Δ^8
3								
7	4							
9	2	9						
4	6	4	6					
5	1	6	2	7				
1	7	6	0	9	2			
8	7	0	5	5	7	5		
3	6	10	10	5	0	4	10	
2	10	4	5	6	1	1	8	9

Page 52.

For this purpose we will count the difference Δ^8 of constant (in this case $\Delta^8=9$). This is implemented by procedure, reverse to the calculation of finite differences, for example, for the new value Δ^7 we will have $8+9 \pmod{11}=6$.

Let us write out the latter/last line of differences and let us use to it the procedure of the continuation of table.

As a result initial code 3, 7, 9, 4, 5, 1, 8, 3, 2 is supplemented by two new positions: 2, 0; subsequently all symbols of the code periodically are repeated. This expanded code we will consider as surplus. Let us introduce distortion, for example, into third position of the code in question, after replacing symbol 9 with symbol 5. Let us compose for distorted code 3, 7, 5, 4, 5, 1, 8, 3, 2, 2, 0 table of differences (taking into account to periodicity). We will obtain:

	$\bar{f}$	$\bar{\Delta}$	$\bar{\Delta}^2$	$\bar{\Delta}^3$	$\bar{\Delta}^4$	$\bar{\Delta}^5$	$\bar{\Delta}^6$	$\bar{\Delta}^7$	$\bar{\Delta}^8$	$\bar{\Delta}^9$	$\bar{\Delta}^{10}$
(1) Информационные символы	3	3	5	7	10	10	2	2	9	4	7
	7	4	1	7	0	1	2	0	9	0	7
	5	9	5	4	8	8	7	5	5	7	7
	4	10	1	7	3	6	9	2	8	3	7
	5	1	2	1	5	2	7	9	7	10	7
	1	7	6	4	3	9	7	0	2	6	7
	8	7	0	5	1	9	0	4	4	2	7
	3	6	10	10	5	4	6	6	2	9	7
	2	10	4	5	6	1	8	2	7	5	7
(2) Избыточные символы	2	0	1	8	3	8	7	10	8	1	7
	0	9	9	8	0	8	0	4	5	8	7
	3	3	5	7	10	10	2	2	9	4	7
	7	4	1	7	0	1	2	0	9	0	7

Key: (1). Informational symbols. (2). Surplus symbols.

Column of differences Δ^8 - control room. The propagation of the error in the table of finite differences is contoured.

2	10	4	5	6	1	1	8	9
2	0	1	8	3	8	7	6	9
0	9	9	8	0	8	0	4	9
3	3	5	7	10	10	2	2	9
7	4	1	7	0	1	2	0	9
9	2	9	8	1	1	0	9	9
4	6	4	6	9	8	7	7	9
5	1	6	2	7	9	1	5	9
1	7	6	0	9	2	4	3	9
8	7	0	5	5	7	5	1	9
3	6	10	10	5	0	4	10	9
2	10	4	5	6	1	1	8	9

Page 53.

From the analysis of this table it is evident that it possesses the discovering and correcting ability: distortion in the control column is the sign/criterion of error; the distorted part of this column is limited to the pairs of the undistorted symbols; the horizontal outline of the distorted part of the table of finite differences indicates the distorted position of the initial code. Correction can be carried out in the undistorted line, which directly adjoins that distorted:

7	4	1	7	0	1	2	0	9
9	2	9	8	1	1	0	9	9

Based on this example is shown the possibility in principle of using the methods of self correction, used in the theory of interpolation, for the construction of the self-correcting codes. However, the construction of the tables of finite differences especially at the high values of p and the survey/coverage of these tables - procedure is bulky. In order not to enumerate the table of differences, we will use the structure of the formula of a finite difference in the m order for the sequence of numbers $a_0, a_1, \ldots, a_m$:

$$\Delta^m a_0 = \sum_{k=0}^{m} (-1)^{m-k} \binom{m}{k} a_k.$$

Let G(p) - be the prime field of characteristic p, $\varphi(\psi)$ — the function, determined on G(p) with the values in G(p). Let us consider finite differences $|\Delta^m \varphi(\bar{0})|_p$ at some values of m. Since

$$\binom{p}{r} \equiv \begin{cases} 0, & \text{при}^{(1)}\ r \neq 0 \\ 1, & \text{при}\ r = 0,\ p \end{cases} \pmod{p},$$

Key: (1). with

that

$$\Delta^p \cdot \varphi(0) = \varphi(0) - \varphi(|p|_p) = 0.$$

By this is explained the periodicity of the tables of finite differences. Further, since

$$\binom{p-s}{r} = \frac{(p-s)(p-s-1)\ldots(p-s-r+1)}{r!},$$

with $s \geqslant 1$

$$\binom{p-s}{r} \equiv (-1)^r \binom{s+r-1}{r} \pmod{p}.$$

Page 54.

Hence, in particular

$$|\Delta^{p-1}\varphi(0)|_p = \left|\sum_{k=0}^{p-1}\varphi(k)\right|_p,$$

$$|\Delta^{p-2}\varphi(0)|_p = |-\sum_{k=0}^{p-2}(k+1)\varphi(k)|_p.$$

By construction of the self-correcting code on these differences are superimposed the limitations

$$|\Delta^{p-2}\varphi(0)|_p = 0, \quad |\Delta^{p-1}\varphi(0)|_p = 0.$$

These limitations lead to the following formulas of the calculation/enumeration of surplus symbols of the code:

$$\alpha_{p-2} = |\varphi(p-2)|_p = |\sum_{k=0}^{p-3}(k+1)\varphi(k)|_p,$$

$$\alpha_{p-1} = |-\varphi(p-1)-\varphi(p-2)|_p = |\sum_{k=0}^{p-3}\varphi(k)|_p,$$

which are simultaneously checkout relationships/ratios and from which it follows that the code indicated corrects any error for arbitrary symbol. Actually/really, let the error occur in the informational symbol with number k=i ($0 \leqslant i \leqslant p-3$), then for the distorted code we will have

$$\tilde{\alpha}_{p-2} = |\sum_{k=0}^{p-3}(k+1)\varphi(k)+(i+1)\tilde{\varphi}(i)|_p,$$

$$\tilde{\alpha}_{p-1} = |\sum_{k=0}^{p-3}\varphi(k)+\tilde{\varphi}(i)|_p.$$

where $\tilde{\varphi}(i) = |\varphi(i) - \nabla_i|_p$.

We form the differences

$$\zeta_{p-1} = |\alpha_{p-1} - \tilde{\alpha}_{p-1}|_p = |-\nabla_i|_p. \qquad (1)$$

$$\zeta_{p-2} = |\alpha_{p-2} - \tilde{\alpha}_{p-2}|_p = |(i+1)(-\nabla_i)|_p. \qquad (2)$$

Page 55.

From relationships/ratios (1) and (2) it follows that for arbitrary error $\nabla_i \neq 0$ occur relationships/ratios $\zeta_{p-1} \neq 0$, $\zeta_{p-2} \neq 0$, moreover ζ_{p-1} is the value of correction, and the number of the distorted position is determined from (2) according to the formula

$$i = ||\varphi_{p-1}^{-1} \cdot \varphi_{p-2}|_p - 1|_p.$$

If distortion occurs on one of surplus positions, then one element/cell of pair $(\zeta_{p-2}, \zeta_{p-1})$ is different from zero and precisely on that position on which occurred the distortion, moreover this element/cell is the value of correction.

Thus, in all cases isolated error unambiguously is discovered and is corrected.

Example. Let p=11. Let be given code (3, 7, 9, 4, 5, 1, 8, 3, 2). Let us form surplus symbols:

$$a_9 = |3+2\cdot 7+3\cdot 9+4\cdot 4+5\cdot 5+6\cdot 1+7\cdot 8+8\cdot 3+9\cdot 2|_{11} = 2,$$

$$a_{10} = |3+7+9+4+5+1+8+3+2|_{11} = 9.$$

Let the distortion be is permitted on the position with number i=2: the symbol of 9 distorted to 5. We form surplus symbols of the code

$$\tilde{a}_9 = |3+2\cdot7+3\cdot5+4\cdot4+5\cdot5+6\cdot1+7\cdot8+8\cdot3+9\cdot2|_{11}=1,$$

$$\tilde{a}_{10} = |3+7+5+4+5+1+8+3+2|_{11}=5.$$

Hence

$$\zeta_9 = |a_9-\tilde{a}_9|_{11} = |2-1|_{11}=1,$$

$$\zeta_{10} = |a_{10}-\tilde{a}_{10}|_{11} = |9-5|_{11}=4.$$

Consequently,

$$i = |4^{-1}\cdot1-1|_{11} = |3-1|_{11}=2,$$

i.e. symbol with number i=2 must be corrected by addition to value 4.

Let us consider more general/more common/more total approach to the construction of the codes, which discover and which correct errors, that is based on the nonpositional multiplicative composition of vectors and using representations of polynomials in the form of Lagrange's interpolation formulas. In connection with this the self-correcting codes are named (n, k) Lagrange's -codes. Are traced also the design features of the synthesis of Lagrange's self-correcting codes.

Page 56.

(N, k) Lagrange's -codes.

In the theory of coding the linear code, which corrects errors, is interpreted as linear subspace E_m (dimension m) linear space E_n (m<n) above final field F_q. Code vector $\bar{a}=(\alpha_1, \alpha_2, \ldots, \alpha_n)$ is called correct, if $\bar{a} \in E_m$. Are examined those combinations of errors $\bar{\delta}$ vector - error), which derive/conclude the distorted vector from E_m, i.e. $\bar{a}+\bar{\delta} \not\subset E_m$. The procedure of detection and correction of errors or, which is the same thing, the algorithms of coding and decoding, depend on the method of introduction of multiplicative composition $\bar{a}*\bar{b}$ above the vectors of space E_n. The assignment to the multiplicative operation *, associative and distributive relative to additive operation + about that satisfying the limitation

$$\forall \alpha \in F_q,\ x,\ y \in E_n: \alpha x*y = x*\alpha y = \alpha(x*y),$$

converts space E_n into the linear algebra (rank n above F_q).

Let us consider the following type of the composition of the code vectors of the linear code.

$$\bar{a}=(\alpha_1, \alpha_2, \ldots, \alpha_n), \quad \bar{b}=(\beta_1, \beta_2, \ldots, \beta_n).$$

then

$$\bar{a}*\bar{b}=(\alpha_1\beta_1, \alpha_2\beta_2, \ldots, \alpha_n\beta_n). \qquad (3)$$

Relative to this composition in work [1] it is said: "Determined thus multiplication it is used is sufficiently rare". Are known only Reed-Muller's codes [1], which are based on the composition indicated.

Let us introduce the following interpretation of the vectors of linear space E_n. Then $\omega_1, \omega_2, \ldots, \omega_n$ - n of different elements/cells of field $F_q (n \leqslant q)$, of those regulated somehow.

Under vector $\bar{a}=(\alpha_1, \alpha_2, \ldots, \alpha_n)$ is understood polynomial a(x), which at points $\omega_1, \omega_2, \ldots, \omega_n$ takes values $\alpha_1, \alpha_2, \ldots, \alpha_n$ respectively. Then under composition $\overline{a*b}$ of vectors $\bar{a}=(\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\bar{b}=(\beta_1, \beta_2, \ldots, \beta_n)$ is understood the polynomial, which accepts at points $\omega_1, \omega_2, \ldots, \omega_n$ of value $\alpha_1\beta_1, \alpha_2\beta_2, \ldots, \alpha_n\beta_n$.

The mathematical apparatus of the synthesis of the self-correcting codes, which is based on composition (3), is tight connection with apparatus of the theory of interpolation in the final fields. In connection with this let us note some most important properties of this apparatus.

Page 57.

Let $F_q[x]$ - be ring of the polynomials above field F_q. In view of the algorithm of the Euclidean division of polynomials for any pair of polynomials f(x), p(x) $\in F_q[x]$ (st. p(x)$\neq$0) there is an only pair of polynomials q(x), r(x) $\in F_q[x]$, of such, that

$$f(x)=q(x)\cdot p(x)+r(x), \quad (4)$$

$$\text{ст. } r(x)<\text{ст. } p(x). \quad (5)$$

Let us designate through $<\cdot|_{p(x)}$ many all polynomials of degree less than degree p(x), and

$$r(x)=<f(x)|_{p(x)}, \quad (6)$$

$$q(x)=\left[\frac{f(x)}{p(x)}\right]=\frac{f(x)-<f(x)|_{p(x)}}{p(x)}. \quad (7)$$

It is not difficult to check the validity of the following assertion.

Theorem 1. Expansion (4) under condition (5) determines representation $<f(x)|_{p(x)}$ of ring $F_q[x]$ onto ring $<\cdot|_{p(x)}$, i.e.

$$<f(x)|_{p(x)}: F_q[x]\to<\cdot|_{p(x)}.$$

This representation possesses the properties:

1. $\ll f(x)|_{p(x)}+<q(x)|_{p(x)}|_{p(x)}=<f(x)|_{p(x)}+<q(x)|_{p(x)}$;
2. $\ll f(x)|_{p(x)}\cdot<q(x)|_{p(x)}|_{p(x)}=<f(x)\cdot q(x)|_{p(x)}$;
3. $\ll f(x)|_{p(x)q(x)}|_{p(x)}=<f(x)|_{p(x)}$;
4. $\left[\frac{f(x)}{p(x)}\right]=0$, если $<f(x)|_{p(x)}=f(x)$;
5. $<q(x)f(x)|_{p(x)q(x)}=q(x)<f(x)|_{p(x)}$;
6. $<\left[\frac{f(x)}{p(x)}\right]\Big|_{q(x)}=<\left[\frac{<f(x)|_{p(x)q(x)}}{p(x)}\right]\Big|_{q(x)}$.

Using a Euclidean algorithm of division, it is easy to show that, if $\alpha_1, \alpha_2, \ldots, \alpha_n$ - different roots of polynomial $f(x) \in F_q[x]$, the f(x) is divided into $(x-\alpha_1)(x-\alpha_2)\ldots(x-\alpha_n)$. In particular, hence it follows that polynomial $f(x) \in F_q[x]$ of degree n has larger n of roots.

Let $\theta(x)=x^q-x$. This polynomial possesses the important property: it is equal to zero at any values $x \in F_q$.

Page 58.

By zero polynomial, as is known, is called the polynomial whose all coefficients are equal to zero. The ring of polynomials $F_q[x]$ above the final field differs significantly from the ring of polynomials on in terms of the field of characteristic by 0 fact that in $F_q[x]$ are nonzero polynomials, which take value, equal to zero, at all points of field. It is obvious that all these polynomials are multiple $\theta(x)$. Therefore in ring $F_q[x]=<\cdot|_{\theta(x)}$ similar to the case of the field of the characteristic of 0 value, they are equal to zero, at all values $x \in F_q$ accepts only zero polynomial. Hence follows assertion.

Theorem 2. With $n \leqslant q-1$ there is a single polynomial $f(x) \in F_q[x]$ degree $\leqslant n$, such, that he accepts with n+1 the different values the variable/alternating x: $\omega_0, \omega_1, \ldots, \omega_n \in F_q$, the preset values $f(\omega_0)$, $f(\omega_1)$, ..., $f(\omega_n)$. This polynomial is determined by Lagrange's interpolation formula

$$f(x)=\sum_{k=0}^{n} f(\omega_k) L_n^{(k)}(x),$$

where

$$L_n^{(k)}(x)=\frac{(x-\omega_0)(x-\omega_1)\cdots(x-\omega_{k-1})(x-\omega_{k+1})\cdots(x-\omega_n)}{(\omega_k-\omega_0)(\omega_k-\omega_1)\cdots(\omega_k-\omega_{k-1})(\omega_k-\omega_{k+1})\cdots(\omega_k-\omega_n)}.$$

Principle of the introduction to redundancy. Evaluation of Hemming's minimum weight.

Let us introduce auxiliary symbolism.

I_q - many all elements/cells of field F_q, regulated it by the form: $\omega_0<\omega_1<\ldots<\omega_{q-1}$. All subsets of set I_q are regulated in accordance with order I_q.

I_n - subset from the n elements of set I_q.

I_k - subset from k elements of set I_n.

cI_k - complement of a set I_k to I_n.

$$P_{I_k}(x)=\prod_{\omega_s\in I_k}(x-\omega_s),\ P_{I_k}^{(r)}(x)=\frac{P_{I_k}(x)}{x-\omega_s},\quad L_{I_k}^{(r)}(x)=\frac{P_{I_k}^{(r)}(x)}{P_{I_k}^{(r)}(\omega_r)},\ \omega_r\in I_k.$$

In accordance with interpretation examined above of vectors

$(\alpha_1, \alpha_2, \ldots, \alpha_n)$ space E_n is equivalent to the ring

$$<\cdot|_{P_{I_n}(x)} = \{a(x) | a(x) = \sum_{\omega_r \in I_n} \alpha_r L_{I_n}^{(r)}(x)\}.$$

Page 59.

The circular operations above the vectors are determined by the rules:

$$\bar{a} + \bar{b} = (\alpha_1 + \beta_1, \alpha_2 + \beta_2, \ldots, \alpha_n + \beta_n) \longleftrightarrow a(x) + b(x) = $$
$$= \sum_{\omega_r \in I_n} (\alpha_r + \beta_r) L_{I_n}^{(r)}(x).$$
$$\bar{a} \times \bar{b} = (\alpha_1 \cdot \beta_1, \alpha_2 \cdot \beta_2, \ldots, \alpha_n \cdot \beta_n) \longleftrightarrow < a(x) b(x)|_{P_{I_n}(x)} =$$
$$= \sum_{\omega_r \in I_n} \alpha_r \beta_r L_{I_n}^{(r)}(x).$$

Let us introduce two types of the conversion of code words.

1. Interpolation from assemblies J to assemblies cJ. The entity of conversion is reduced to the following. Let $J = (i_1, I_2, \ldots, i_k)$, on code vector $(\alpha_{i_1}, \alpha_{i_2}, \ldots, \alpha_{i_k})$ be restored Lagrange's multi-canoe

$$a_r(x) = < a(x)|_{P_J} = \sum_{\omega_{i_s} \in J} \alpha_{i_s} L_J^{(s)}(x).$$

Further in assemblies $\omega_j \in cJ = J_n/J$ are calculated the values of polynomial $a_r(x)$:

$$a_J(\omega_j) = \sum_{\omega_{i_s} \in J} \alpha_{i_s} L_J^{(s)}(\omega_j) \quad \forall \omega_j \in cJ.$$

2. Taking greatest integer from assemblies I_n to assemblies J.

The entity of conversion is reduced to the following. In assemblies J are calculated the values of the polynomial

$$\left[\frac{<a(x)|_{P_{I_n}(x)}}{P_{cJ}(x)}\right].$$

Is realized this conversion by means of the interpolation in accordance with formula (7).

Determination. I_k - many informational assemblies with cI_k - many surplus assemblies. $a_1, a_2, \ldots, a_k$ - information symbols of code vector. Surplus symbols $a_{k+1}, \ldots, a_n$ let us determine by interpolation from assemblies I_k to the assemblies with cI_k according to Lagrange's formula

$$a_{k+s}=\sum_{\omega_i\in I_k} a_i L^{(i)}_{I_k}(\omega_{k+s}) \quad 1\leqslant s\leqslant n-k.$$

Page 60.

As result we obtain (n, k)-code which we will call Lagrange's code.

From the determination escape/ensues the following.

1. Code vector $<a(x)|_{P_{I_n}}\sim(a_1, a_2, \ldots, a_n)$ belongs to code space $E_k\sim<\cdot|_{P_{I_k}}$ when and only when $<a(x)|_{P_{I_k}}=a(x)$, or, which is the same thing,

$$\left[\frac{<a(x)|_{P_{I_k}}}{P_{I_k}(x)}\right]=0.$$

2. Process of coding does not distcrt informational symbols and is reduced to interpolation frcm irfcrmational assemblies to surplus ones.

3. Any set from k symbols (n, k) of Lagrange's -codes (in view cf theorem 2) can be accepted as set cf informational symbols.

Theorem 3. p(x) and q(x) - mutually simple polynomials. With any $\beta(x)\in <\cdot|_{q(x)}$. the equation

$$\left[\frac{\zeta(x)q(x)}{p(x)}\right]=\beta(x) \qquad (8)$$

has unique solution in class $<\cdot|_{p(x)}$, if st. q(x) $\geqslant$ st. p(x).

Equation (8) is equivalent tc the equality

$$\zeta(x)q(x)-<\zeta(x)q(x)|_{p(x)}=\beta(x)p(x).$$

Hence

$$\ll\zeta(x)q(x)|_{p(x)}|_{q(x)}=<-\beta(x)p(x)|_{q(x)}. \qquad (9)$$

From the condition st. q(x) $\geqslant$ st. p(x) it follows that $<\cdot|_{p(x)}\subseteq<\cdot|_{q(x)}$, therefore $\ll\zeta(x)q(x)|_{p(x)}|_{q(x)}=<\zeta(x)q(x)|_{p(x)}$ and equality (9) accepts the form

$$<\zeta(x)q(x)|_{p(x)}=<-\beta(x)p(x)|_{q(x)}.$$

Hence

$$\zeta(x) = \langle q^{-1}(x) \langle -\beta(x) p(x)|_{q(x)}|_{p(x)}.$$

Theorem 4. For the detection of a d- multiple error by Lagrange's code it is necessary and it is sufficient d cf excess symbols.

Sufficiency. Let $\Omega_{I_n}(\leqslant d)$- many all subsets of set I_n, consisting be not more than of d elements/cells. With any $J \in \Omega_{I_n}(\leqslant d)$ to the vector of the error for multiplicity $\leqslant d$ is compared the polynomial

$$\nabla_d(x) = P_{cJ}(x) \cdot \Delta_J(x),$$

where

$$\Delta_J(x) = \sum_{\omega_i \in J} \delta_i P_J^{(i)}(x).$$

Respectively, distorted vector $\tilde{a}(x) = a(x) + \nabla_d(x)$ it will have in the symbols with numbers i $(\omega_i \in J)$ of the distortion

$$\delta_i [P_{I_n}^{(i)}(\omega_i)].$$

Transfer of greatest integer from information assemblies I_k to surplus assemblies with cI_k leads to the expression

$$\beta(x) = \left\langle \left[\frac{\langle \tilde{a}(x)|_{P_{I_k}(x)}}{P_{I_k}(x)} \right] \right|_{P_{cI_k}} = \left\langle \frac{\tilde{a}(x) - \langle \tilde{a}(x)|_{P_{I_k}(x)}}{P_{I_k}(x)} \right|_{P_{cI_k}}.$$

But

$$\tilde{a}(x) - \langle \tilde{a}(x)|_{P_{I_k}} = a(x) + \nabla_d(x) - \langle a(x)|_{P_{I_k}} - \langle \nabla_d(x)|_{P_{I_k}}.$$

Since

$$<a(x)|_{P_{1_k}} = a(x),$$

that

$$\beta(x) = \left[\frac{P_{cJ}(x)\Delta_J(x)}{P_{1_k}(x)}\right], \quad \beta(x) \in <\cdot|_{P_{c1_k}}.$$

So that the latter/last equation would have unique solution with any $\beta(x) \in <\cdot|_{P_{c1_k}}$. according to theorem 3, it is sufficient so that would be satisfied the condition st. $P_{cJ}(x) \geqslant$ ст.$P_J(x)$. But st. $P_J(x) \leqslant d$. Therefore for the detection of any error for multiplicity not higher than d of sufficiency so that would be satisfied the condition st. $P_{c1_k}(x) = d$.

Need. Let us show that the minimum weight of Hemming of Lagrange's code with d by surplus symbols is equal to d+1.

Page 62.

Let code vector $a(x) \longleftrightarrow (a_1, a_2, \ldots, a_n)$ belong to code space, i.e., $<a(x)|_{P_{1_k}} = a(x)$ then st. $a(x) \leqslant n-d-1$, since st. $P_{1_k}(x)=n-d$. Hence it follows that a maximum number of roots of polynomial a(x) is equal n-d-1, i.e., any code vector a(x) has the less than d+1 nonzero symbols.

Corollary 1. As is *known [for 2]*, the code, which has length n, the minimum distance d+1 and a maximally possible number of informational symbols n-d, it is called maximum. Thus, Lagrange's codes with any $n \leqslant q-1$ and any method of the adjustment of assemblies are maximum.

Corollary 2. For detection and correction of d-multiple error (n, k) by Lagrange's -code $n \leqslant q-1$ it suffices to have 2d surplus ones of symbol.

Comparison of Lagrange's codes with Reed- Solomon's codes.

Let us consider the translation algorithm of the polynomial, represented in the form of Lagrange

$$F(x)=\sum_{\omega_i\in I_q}F(\omega_i)L^{(i)}_{I_q}(x). \qquad (10)$$

into the polynomial, arranged/located according to the degrees

$$F(x)=\sum_{k=0}^{q-1}f(k)x^k. \qquad (11)$$

We have

$$L^{(i)}_{I_q}(x)=-\frac{\theta(x)}{x-\omega_i}=-\Phi\left(\frac{x}{\omega_i}\right), \qquad (12)$$

where

$$\Phi(x)=\frac{x^q-x}{x-1}=\sum_{k=1}^{q-1}x^k. \qquad (13)$$

Formula (13) makes it possible to present Lagrange's fundamental polynomials (12) in the form of polynomial according to degrees of x. Let us assume $\omega_0=0$, $\omega_1=1$ (unit of field). Substituting exponential representation $L^{(i)}_{I_q}(x)$ in (10) and being congruent/equating that obtained with (11), we have

$$f(0)=F(\omega_0),$$

$$f(k)=-\sum_{j=1}^{q-1}F(\omega_j)\omega_j^{-k}, \quad 1<k<q-2,$$

$$f(q-1)=-\sum_{j=0}^{q-1}F(\omega_j).$$

Page 63.

In particular, examining many assemblies I'_q with the excluded zero element/cell, we obtain, that the polynomial

$$F(x)=\sum_{\omega_j\in I'_q} F(\omega_j)L_{I'_q}^{(j)}(x) \quad (14)$$

is converted into the polynomial

$$F(x)=\sum_{k=0}^{q-2} f(k)x^k \quad (15)$$

according to the formulas

$$f(x)=-\sum_{j=1}^{q-1} F(\omega_j)\omega_j^{-k}, \quad 0\leqslant k\leqslant q-2. \quad (16)$$

Formulas (15) and (16), strictly, and determine coding by Reed-Solomon's codes.

$f_0, f_1, \ldots, f_{k-1}$ - informational symbols. We form the polynomial

$$F(x)=f_0+f_1x+f_2x^2+\ldots+f_{k-1}x^{k-1}$$

and let us form the values of polynomial F(x) in assemblies $\omega_i\in I'_q$: $F(\omega_1), F(\omega_2), \ldots, F(\omega_{q-1})$, which determine code vector. The process of decoding is reduced to the calculation of sums (16) with $0\leqslant k\leqslant q-2$, in this case as checkout relationships/ratios they serve

$$\sum_{j=1}^{q-1} F(\omega_j)\omega_j^{-(k+s)}=0, \quad 0\leqslant s\leqslant q-k-2.$$

Thus, Reed-Solomon's codes use for the process of coding and decoding of formula (15) and (16), that escape/ensue from Lagrange's formula (14). Differ Reed-Solomon's codes from Lagrange's codes in terms of the processes of coding and decoding: in the case of Lagrange's codes informational symbols $f_0, f_1, \ldots, f_{k-1}$ remain invariant in the recording of code vector, whereas in the case of Reed-Solomon's codes they are converted into the values of polynomial P(x) at points $\omega_1, \omega_2, \ldots, \omega_k$.

Page 64.

This fact can serve as basis for the decrease of the capacity of conversions in the processes of coding and decoding in the case of Lagrange's codes in the comparison with Reed-Solomon's codes.

A vital difference in Lagrange's codes from the cyclic codes escape/ensues from a difference in the types of the multiplicative composition of the vectors on which, strictly, rest the methods of coding and decoding. It lies in the fact that Lagrange's codes can be considered as the codes, in a sense dual by cyclic.

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Page 65.

ONE METHOD OF DETECTION AND CORRECTION OF THE ERRORS IN A SYSTEM OF RESIDUAL CLASSES.

V. M. Amerbayev, Yu. L. Zakharov.

During the construction of the contemporary computers, which work on real time with the high degree of reliability, the use/application of the corrective codes for the correction of the errors in transmittings and information processing becomes urgent practical task. If the corrective codes sufficiently extensively are used for the protection from the disturbances with the transmission of information, its reading from different carriers, then the use of the corrective arithmetic codes is limited. This is explained by the fact that very process of decoding (detection of error) takes away the time, commensurated with the time of the fulfillment of arithmetic operation, if we for the correction of errors apply the divisible and indivisible arithmetic AN-codes or (n, k)-codes. The decoding equipment for these codes is obtained sufficiently simple, if we use them for the protection from the short duration failures in the devices/equipment of consecutive information processing. However, in the devices/equipment of parallel information processing their use/application leads to a sharp increase in the equipment for decoding and a decrease of the high speed of arithmetic unit. The protection of very diagrams of decoding requires in this case of special measures (at least redundancy) so that the failures of elements/cells would not upset the operation of computer. Therefore the given codes are used in the digital computers (TsVM) only for the detection of errors. Is known one additional class of the corrective arithmetic codes - codes in the system of residual classes (SOK) [3].

The introduction of two control bases/bases for the codes in the remainders/residues during the specific limitations makes it possible to discover and to correct all isolated errors from the working or control basis of system.

Page 66.

However, the method of determining the inaccurate deduction by omission of one of the basis of system and the restoration/reduction of number value in that abbreviated/reduced SOK requires, in the first place, the long computing time, in the second place, of a considerable quantity of constants with the diverse variants of the excluded bases/bases. For the high speed arithmetic units this method is not sufficiently to efficient ones.

Are at the present time known works [5, 6], dedicated to the analysis of the properties of the self correction of the code in the remainders/residues. From the works about the concrete/specific/actual realization of the properties of the self correction of the codes in the remainders/residues let us note work [4], where is examined the four-modular version of numeration system in the remainders/residues and is shown the high efficiency of the properties of the self correction of the codes indicated.

The procedure of decoding it is expedient to combine with the operation of rounding. This will make it possible, in the first place, to avoid the danger of the multiplication of the error in the process of rounding, in the second place, to avoid the effect of the processes of decoding and check on the productivity AU, thirdly, to implement the process of decoding on the equipment for rounding. This joining of the procedure of decoding to the process of rounding requires the account of effect on VA of the self correction of all modifications, introduced into the device/equipment of rounding for the purpose of the decrease of equipment and increase in the high speed.

We study the properties of the self correction of the code in the remainders/residues, being based on the procedure of rounding, examined in work [8]. Let us note that in this article is used the notation, accepted in [8].

Let the operating range, determined by the system of bases/bases P, be connected with the further range, determined by basis/base Q, with relationship/ratio P=Q-h.

Let, further, K_1, K_2 - control bases/bases, utilized for the detection of error, determinations of its position and value. Let us designate through P'' value P''-2tP, where basis/base t is dictated by the procedure of rounding, and basis/base 2 is used for the sign representation of numbers [7]. Let us designate through p'_1 the first basis of system P, and through p_1 - value $p_1=2tp'_1$.

Let the error occur on one of the basis of system P''=2tP. The true value of number A can be presented in the form

$$|A^-|^+_{P''QK_1K_2}=C^-+\tilde{D}\cdot P.$$

Let us look, what effect proves to be the imposition of error on the i basis/base range P''.

Page 67.

It is here necessary to distinguish two cases: i=1 and i≠1, since in the first case of p_1 - even number ($p_1=2tp'_1$). The imposition of error indicates addition and to initial A of a number of form

$\Delta_i P_i''QK_1K_2$, where Δ_i - value of error on i basis/base ($1 \leqslant \Delta_i \leqslant p_i - 1$)

$$P_i = \frac{P''}{p_i}; \quad P'' = 2 \cdot t \cdot P.$$

In this case the distorted number will take the following form:

$$\|A^-|^+_{P''QK_1K_2} + \Delta_i P_i''QK_1K_2|_{P''QK_1K_2} = |C^+ + \Delta_i P_i''QK_1K_2 + + \widetilde{D}^+ \cdot P|_{P''QK_1K_2}. \quad (1)$$

Here and throughout

$$\widetilde{P} = \frac{P''}{p_1}; \quad P_i = \frac{P}{p_i}; \quad p_1' = \frac{p_1}{2t}.$$

Taking into account the relationship/ratio between ranges (P+h=Q), the value of error it is possible to reduce to the form

$$\Delta_i P_i''QK_1K_2 = \Delta_i P_i''K_1K_2h + \Delta_i P_i''K_1K_2P = |t\Delta_i K_1K_2h|^{\bullet}_{p_i} \cdot P_i + + \left(\left[\frac{t\Delta_i K_1K_2h}{p_i}\right] + \Delta_i P_i''K_1K_2\right) \cdot P \quad (\overset{(1)}{\text{при}}\ i \neq 1)$$

Key: (1). with.

and

$$\Delta_1 \widetilde{P}QK_1K_2 = \Delta_1 \widetilde{P}K_1K_2h + \Delta_1 \widetilde{P}K_1K_2P = |\Delta_1 K_1K_2h|_{p_1'} \cdot \widetilde{P} + + \left(\left[\frac{\Delta_1 K_1K_2h}{p_1}\right] + \Delta_1 \widetilde{P}K_1K_2\right)P \quad (\overset{(1)}{\text{при}}\ i = 1).$$

Key: (1). with.

Then expression (1) with i=1 will take the following form:

$$\|A^-|^+_{P''QK_1K_2} + \Delta_1 \widetilde{P}QK_1K_2|^+_{P''QK_1K_2} = \|C^+ + |\Delta_1 K_1K_2h|_{p_1'} \cdot \widetilde{P}|_P - - \frac{P}{2} + \left[\frac{t\Delta_1 K_1K_2h}{p_1}\right] + e_1 + \Delta_1 \widetilde{P}K_1K_2 + \widetilde{D}|_{tQK_1K_2} \cdot P|_{P''QK_1K_2},$$

and with i=1

$$\|A^-|^+_{P''QK_1K_2} + \Delta_i P_i''QK_1K_2|^+_{P''QK_1K_2} = \|C^+ + |t\Delta_i K_1K_2h|_{p_i} \cdot P_i|^+_P - - \frac{P}{2} + \left[\frac{t\Delta_i K_1K_2h}{p_i}\right] + e_1 + \Delta_i P_i''K_1K_2 + \widetilde{D}\Big|^+_{tQK_1K_2} \cdot P|_{P''QK_1K_2},$$

where

$$C^- = C^+ - \frac{P}{2};\quad 1 \leqslant \Delta_i \leqslant p_i - 1;\quad 1 \leqslant \Delta_1 \leqslant p_1 - 1;$$

$$\varepsilon_1 = \left[\frac{C^+ + |t\Delta_i K_1 K_2 h|_{p_i} P_i}{P}\right];\quad \varepsilon_1 = 0,1.$$

Page 68.

Thus, the distortion of the rounded-off value $\widetilde{D}$ during the expansion of the number, which has isolated error on one of the bases/bases from number P, which follows:

$$\underline{\widetilde{D}} = |\widetilde{D} + \varepsilon_1 + \left[\frac{t\Delta_i K_1 K_2 h}{p_i}\right] + \Delta_i P_i' K_1 K_2|^+_{tQK_1K_2}. \qquad (2)$$

here and throughout $\underline{\widetilde{D}}$ - distorted value $\widetilde{D}$.

The same error itself introduces distortion into the formation of rounded off value $\overline{N}^+$. Representing number A^- in its disintegration in terms of range Q: $A = M^+ + \overline{N}^+ \cdot Q$, we will obtain that the distorted value of value $\overline{N}^+$ will be equally

$$\underline{N}^+ = |\overline{N}^+ + \Delta_i P_i'' K_1 K_2|^+_{tPK_1K_2}. \qquad (3)$$

The parameter δ^+, depending on values $\widetilde{D}^+$ and $\overline{N}^+$, will obtain in this case the following distortion:

$$\underline{\delta}^+ = |\underline{\widetilde{D}}^+ - \underline{N}^+|_t^+ = |\delta^+ + \varepsilon_1 + \left[\frac{t\Delta_i K_1 K_2 h}{p_i}\right]|_t^+. \qquad (4)$$

Let us designate value $\widetilde{D}^+$, which is n. n. V. on mod tQK_1K_2, by symbol D^+_{np}, and corrected value $|\overline{N}^+ + \delta^+|^+_{tPK_1K_2}$ - by symbol D^+_{no} and we form their difference in control tases/bases (ξ). Let us show in this case that $|\xi|_t^+=0$. Actually/really,

$$|\xi|_t^+=|D^+_{\text{пр}}-D^+_{\text{исп}}|_t^+=||\tilde{D}+e_1+\left[\frac{t\Delta_i K_1 K_2 h}{p_i}\right]+\Delta_i P_i'' K_1 K_2|_t^+-$$

$$-|\overline{N}+\Delta_i P_i'' K_1 K_2+\underline{\delta}^+|_t|_t=|\delta^++e_1+\left[\frac{t\Delta_i K_1 K_2 h}{p_i}\right]-\underline{\delta}^+|_t;$$

but

$$\delta^++e_1+\left[\frac{t\Delta_i K_1 K_2 h}{p_i}\right]=\underline{\delta}^+,$$

consequently,

$$|\xi|_t^+=0.$$

Page 69.

Let us determine the form of a difference in the distorted rounded-off values in the case of isolated error in basis/base p_i, after substituting into the formula for a difference in value (2), (3), (4)

$$\xi=|D^+_{\text{пр}}-D^+_{\text{исп}}|_{tK_1K_2}+||\tilde{D}+e_1+\left[\frac{t\Delta_i K_1 K_2 h}{p_i}\right]+\Delta_i P_i'' K_1 K_2|^+_{tK_1K_2}-$$

$$-|\overline{N}^++\Delta_i P_i'' K_1 K_2|^+_{tK_1K_2}-\underline{\delta}^+|_{tK_1K_2}=||\delta^+|_{tK_1K_2}+\left[\frac{t\Delta_i K_1 K_2 h}{p_i}\right]+$$

$$+e_1-\underline{\delta}^+|^+_{tK_1K_2}.$$

If from the latter/last expression to isolate $|\underline{\delta}^+|_t$, the final value of a difference in the rounded-off values will take the form

$$\xi=t\left(\left[\frac{\Delta_i K_1 K_2 h}{p_i}\right]+\rho_{\text{пр}}\right), \qquad (5)$$

where

$$\rho_{\text{пр}}=\left[\frac{\delta^++e_1+\left|\left[\frac{t\Delta_i K_1 K_2 h}{p_i}\right]\right|^+_t}{t}\right].$$

Parameter ρ_{np} characterizes by itself the spread of difference, connected with rounding and imposition of error on the true value of a number. Let us consider the borders of its change. Since

$$0 \leqslant \delta^+ \leqslant t-1,\ \varepsilon_1=0,1,$$

$$0 \leqslant \lambda = \left|\left[\frac{t\Delta_i K_1 K_2 h}{Pt}\right]\right|_t \leqslant t-1,$$

that ρ_{np} can take value of 0. 1.

Let number A be is distorted on one of the bases/bases q_j from the system of bases/bases Q. Let us carry out the analogous analysis of the scatter of a difference in the rounded off values. During the expansion with Q on $P''\ K_1K_2$ correct number will be distorted as follows:

$$\|A\rceil^+_{P''QK_1K_2} + \Delta_j P'' Q_j K_1 K_2|^+_{P''QK_1K_2} = \|A\rceil^+_Q - \frac{P}{2} + \\ + \Delta_j P'' Q_j K_1 K_2 + (N^+ + \eta_B) Q|^+_{P''QK_1K_2}; \qquad (6)$$

$$\eta_B = \left[\frac{|A\rceil^+_Q + \frac{P}{2}}{Q}\right].$$

Page 70.

Taking into account that P=Q-h, the value of error let us present in the form

$$\Delta_j P'' Q_j K_1 K_2 = -t\Delta_j K_1 K_2 h + t\Delta_j K_1 K_2 Q_j Q = (q_j -$$

$$-|t\Delta_j K_1 K_2 h|^+_{q_j}) \cdot Q_j + (-1 - \left[\frac{t\Delta_j K_1 K_2 h}{q_j}\right] + t\Delta_j Q_j K_1 K_2) \cdot Q.$$

Then formula (6) will take the following form:

$$|\underline{A}^-|^+_{P''QK_1K_2} = |||A^-|^+_Q + (q_j - |t\Delta_j K_1 K_2 h|^+_{q_j}) \cdot Q_j|_Q -$$

$$-\frac{P}{2} + |N^+ + \eta_2 + \varepsilon_2 - 1 - \left[\frac{t\Delta_j K_1 K_2 h}{q_j}\right] + t\Delta_j K_1 K_2 Q_j Q|_{P''QK_1K_2}.$$

Here

$$\varepsilon_2 = \left[\frac{||A^-|^+_Q + \frac{P}{2}|_Q + (q_j - |t\Delta_j K_1 K_2 h|^+_{q_j}) \cdot Q_j}{Q}\right]$$

the allowed transition through modulus/module Q during the imposition of error on basis/base q_j

$$\varepsilon_2 = 0,1.$$

If one considers that during the expansion with Q on $P''K_1K_2$ is used the inaccurate value of rank Δ_m, and also shift/shear for the constant for the formation of the correction of rounding δ^+ in the class N.N.V. on mod t, value $\underline{\overline{N}}^+$ will take the form

$$\underline{\overline{N}}^+ = |\overline{N}^+ + \theta - \left[\frac{t\Delta_j K_1 K_2 h}{q_j}\right] + t\Delta_j K_1 K_2 Q_j|^+_{P''K_1K_2}.$$

Here $\theta = \eta - \eta^* + \varepsilon_2 - 1$ - indeterminate value, which takes the values

$$-2 \leqslant \theta \leqslant 1. \quad (7)$$

and η, η^* - difference between the exact and inaccurate values of the rank of correct and distorted numbers.

For determining the scatter of difference let us find $\underline{\tilde{D}}$ and $\underline{\delta}^+$ with the error from q_j

$$\|A^{-}|^{+}_{P''QK_1K_2}+\Delta_j Q_j P'' K_1 K_2|^{+}_{P''QK_1K_2}=|C+$$
$$+(\widetilde{D}+t\Delta_j Q_j K_1 K_2)\cdot P|_{P''QK_1K_2}.$$

Page 71.

Consequently,

$$D_{np}=|\widetilde{D}+t\Delta_j Q_j K_1 K_2|_{tQK_1K_2}$$

and

$$\underline{\delta}^{+}=|\underline{\widetilde{D}}-\underline{\overline{N}}^{+}|_t{}^{+}=|\delta^{+}-\theta+\left[\frac{t\Delta_j K_1 K_2 h}{q_j}\right]|_t^{+}\ .$$
$$D^{+}_{\text{nos}}=|\underline{\overline{N}}^{+}+\underline{\delta}^{+}|_{tPK_1K_2}.$$

Let us consider difference in the rounded-off values and scatter of this difference (in this case also $|\xi|_t^{+}=0$):

$$\xi=|D^{+}_{np}-D^{+}_{\text{nos}}|^{+}_{tK_1K_2}=\||\delta^{+}|_{tK_1K_2}-\theta+\left[\frac{t\Delta_j K_1 K_2 h}{q_j}\right]-\underline{\delta}^{+}|_{tK_1K_2}$$

or

$$\xi=t\left(\left[\frac{\Delta_j K_1 K_2 h}{q_j}\right]+\rho_{\text{nos}}\right).$$

where

$$\rho_{\text{nos}}=\left[\frac{\delta^{+}-\theta+\left|\left[\frac{t\Delta_j K_1 K_2 h}{q_j}\right]\right|_t^{+}}{t}\right].$$

The borders of change ρ_{nos} taking into account (7) will be

$$-1\leqslant\rho_{\text{nos}}<2.$$

Finally, if modular error occurs on one of the control bases/bases (for example, on K_1), then

$$\|A^{-}|^{+}_{P''QK_1K_2}+\Delta_{K_1} P'' Q K_2|^{+}_{P''QK_1K_2}=|C+$$
$$+(\widetilde{D}+t\Delta_{K_1} Q K_2)\cdot P|^{+}_{P''QK_1K_2}.$$

This means that the distorted value of the rounded-off value $\underset{\sim}{\widetilde{D}}^{+}$ will be

$$\underline{\widetilde{D}^{+}}=D^{+}_{\text{пр}}=|\widetilde{D}+t\Delta_{K_1}QK_2|_{tQK_1K_2},$$

and

$$\underline{\overline{N}}^{+}=|\overline{N}^{+}+t\Delta_{K_1}PK_2|_{tPK_1K_2}.$$

Page 72.

In this case a difference in the rounding will not obtain distortion, since

$$\underline{\delta}^{+}=|\underline{\widetilde{D}}^{+}-\underline{\overline{N}}^{+}|_{t}{}^{+}=|\widetilde{D}-t\Delta_{K_1}QK_2-\overline{N}^{+}-t\Delta_{K_1}PK_2|_{t}=$$
$$=|\widetilde{D}^{+}-\overline{N}^{+}|_{t}{}^{+}=\delta^{+},$$

since control bases/bases do not accept participation in the formation δ .

Then

$$D^{+}_{\text{зеа}}=|\underline{\overline{N}}^{+}+\delta^{+}|_{tPK_1K_2}$$

and

$$\xi=|D^{+}_{\text{пр}}-D^{+}_{\text{зев}}|_{tK_1K_2}=||\widetilde{D}^{+}-\overline{N}^{+}|_{tK_1K_2}+t\Delta_{K_1}QK_2-$$
$$-t\Delta_{K_1}PK_2-\delta^{+}|_{tK_1K_2}. \qquad (8)$$

After substituting in (8) $|\widetilde{D}^{+}-\overline{N}^{+}|_{t}=\delta^{+}$ and taking into account Q=P+h, we will obtain

$$\xi=t\Delta_{K_1}K_2h.$$

Analogously with the error on basis/base K_2

$$\xi = t\Delta_{K_2} K_1 h.$$

Thus, is obtained the set of the differences which occur with the errors on any of the basis of system P'', Q, K_1, K_2. Their set consists of numbers of form $\xi-1$, ξ, $\xi+1$, $\xi+2$ and is the signs/criteria of the presence of error on one of the basis of the system, accepted for the representation of numbers. If control bases/bases are selected so that each isolated error answers only its set of differences, then isolated errors are divided. In this case control bases/bases make it possible in the process of rounding to check the number, preset by its remainders/residues, i.e., the procedure of decoding and checking of a number does not require the further expenditures of time, while in the case of the detection of the error are uniquely determined both its value and location.

Let us note that the analytical forms of differences ξ can be used for the selection of the control bases/bases K_1, K_2, which satisfy the requirements of the separability of the signs/criteria of errors, obtained by the process of decoding, combined with the process of the nonparallel algorithm of rounding.

Page 73.

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Page 74.

DEPARALLELIZATION OF THE OPERATION OF ROUNDING IN A SYSTEM OF RESIDUAL CLASSES.

V. M. Amerbayev, V. S. Kokorin.

Use of a system of residual classes (SOK) for the coding of numerical information gives the possibility to efficiently unparallelize such operations as addition, subtraction and multiplication, which in connection with this are called modular. The operations of the nonpositional arithmetic whose execution one way or another is connected with the positional representation of the numbers, which consist of several procedures of the type of modular operations, are called nonmodule. The latter include such operations, as the determination of the sign of a number, the overflow of the range of representation, the rounding of a number, the translation of number of SOK into the positional system of numeration, etc. The high speed of nonpositional arithmetic unit to a considerable degree is determined by the length of the execution of the procedure of the rounding of a number, preset in SOK. The known algorithms of the execution of the operation of the rounding of a number, represented of quadratic range [1, 2], consist of two or several procedures of the expansion of the working and surplus ranges, realized especially consecutively/serially.

In connection with this is placed the task of tracing the possibility of the deparallelization of the operation of the rounding of a number in SOK. In this case, in the first place, the artificial form of representation relative to numbers must not affect the operation time of the rounding of number [1, 4], and, in the second place, it is necessary to consider the possibility of using the inaccurate value of rank, which leads to the considerable reduction of the equipment expenditures in the implementation of the procedure of rounding.

Let us clarify the sense of the operation of rounding.

Page 75.

In the article is examined a question of the rounding of the result of the multiplication of the fractions of form A/P, where A - positive integer number, it represented in SOK belongs to range $P=\prod_{i=1}^{n} p_i$; p_i - the basis/base of range P and $i=\overline{1, n}$.

With the multiplication of fractions A_1/P and A_2/P does appear fraction A/P^2, where $A=A_1 \cdot A_2$.

The sense of the operation of rounding consists in converting of fraction A/P^2 to the form $1/P\,[A/P]$, where $[A/P]$ - the near whole, not not exceeding is number A/P.

It is necessary to note that for the purpose of the retention/preservation/maintaining the possibility to implement the operation of multiplication modularly it is necessary to have a redundancy in the representation of numerical information. Let surplus range $Q=\prod_{j=1}^{m} q_j$, $(Q, P)=1$ and $Q=P+h$.

For determining value $[A/P]$ let us present number A in the following form:

$$A=C+D\cdot P, \tag{1}$$

where

$$0<C<P \quad \text{and} \quad 0<A<P^2<P\cdot Q;$$

then

$$\left[\frac{A}{P}\right]=\frac{A-C}{P}=D;$$

since $D<P$, then

$$D=|D|_Q=\left|\frac{A-C}{P}\right|_Q. \tag{2}$$

For calculation $|D|_P$ let us present A in the form

$$A=M+N\cdot Q, \tag{3}$$

where

$$0 \leq M < Q.$$

From expressions (1) and (3) we have

$$\delta = D - N = \frac{M - C + N \cdot h}{P}. \qquad (4)$$

Since $0 \leq M < Q$, $0 \leq C < P$ and $0 \leq N < P$, the correctly following expression:

$$0 \leq \delta \leq h. \qquad (5)$$

Page 76.

Since $0 \leq D < P$, then from expressions (3) and (4) we have

$$D = |D|_P = |N + \delta|_P = \left|\left|\frac{A - M}{Q}\right|_P + \delta\right|_P. \qquad (6)$$

For the determination δ let us introduce further basis/base $P_s > h$, then $\delta = |\delta|_{P_s}$.

Correctly following expression:

$$\delta = |\delta|_{P_s} = \left|\left|\frac{A \cdot h}{P \cdot Q}\right|_{P_s} - \left|\frac{C}{P}\right|_{P_s} + \left|\frac{M}{Q}\right|_{P_s}\right|_{P_s}. \qquad (7)$$

By the specific selection of bases/bases it is possible to select such ranges P and Q so that the value h would be insignificant, in this case formation δ (correction of rounding) in accordance with expression (7) can be realized in parallel with calculation $|N|_P$ and $|D|_Q$. At the termination of determination $|N|_P$ and $|D|_P$ is carried out correction of the rounded-off result on the bases/bases of range P.

Let us pass to the examination of the described algorithm of the rounding of numbers taking into account the sign form of the representation of relative numbers in SOK, given in work [4].

Let A^- - the integer of arbitrary sign, $|A^-|_P^+$ - least non-negative residue (n.n.v.) of number A^- on mod P, and the system of the bases/bases of nonpositional arithmetic is characterized by value $4PQp_g$. For the form of sign in question the result of the rounding of number A^- is value $|D^-|_{4PQp_g}^+$.

From work [4] it is known that the restoration/reduction of relative number B^- according to its representation in SOK $|B^-|_{4P}^+$ is realized in accordance with the expression

$$B^-=\left||B^-|_{4P}^++2P\right|_{4P}^+-2P. \qquad (8)$$

Using (8), let us present number A in the following form:

$$A^-=\left||A^-|_P^++\frac{P}{2}\right|_P^+-\frac{P}{2}+D^-\cdot P. \qquad (9)$$

Since

$$A^-\equiv|A^-|_{4PQp_g}^+ \pmod{4PQp_g}$$

and

$$|A^-|_{4PQp_g}^+=\left||A^-|_{4PQp_g}^+|_P^++\frac{P}{2}\right|_P-\frac{P}{2}+D^+\cdot P, \qquad (10)$$

that

$$D^-\cdot P\equiv D^+\cdot P \pmod{4PQp_g}$$

or

$$D^- \equiv D^+ \pmod{4PQp_g}.$$

Page 77.

With

$$-2P^2 \leqslant A^- < 2P^2 \qquad (11)$$

is correct the inequality

$$-2P \leqslant D^- < 2P,$$

i.e.

$$D^+ = |D^-|^+_{4PQp_g} = |D^-|^+_{4Qp_g} = |D^-|^+_{4Pp_g}. \qquad (12)$$

Consequently, value $|D^-|^+_{4Qp_g}$ is the unknown value of the result of rounding, represented over range $4Qp_g$. For the formation of the deductions of the rounded-off result on the bases/bases of range P let us present number A^- in the form, analogous to expression (9):

$$A^- = ||A^-|^+_{4PQp_g}|_Q + N^- \cdot Q. \qquad (13)$$

Further, since

$$|A^-|^+_{4PQp_g} = ||A^-|^+_{4PQp_g}|_Q + N^+ \cdot Q, \qquad (14)$$

we have

$$N^- \cdot Q \equiv N^+ \cdot Q \pmod{4PQp_g},$$

i.e.

$$N^- \equiv N^+ \pmod{4PQp_g}.$$

Taking into account (10), we will obtain

$$N^+ = |N^-|^+_{4PQp_g} = |N^-|^+_{4Qp_g} = |N^-|^+_{4Pp_g}. \qquad (15)$$

It is obvious that $D^{+}>N^{+}$, then

$$\delta=D^{+}-N^{+} \text{ и}^{(1)} \quad \delta>0. \tag{16}$$

Key: (1). and.

When $p_g>\max\delta$ is correct the inequality

$$\delta=\|D^{-}|^{+}_{p_g}-|N^{-}|^{+}_{p_g}|_{p_g}. \tag{17}$$

From expressions (6), (15), (16) we have

$$|D^{-}|^{+}_{p}=\|N^{-}|^{+}_{p}+\delta|^{+}_{p}. \tag{18}$$

Let us determine the maximum value δ with the rounding of relative numbers.

Page 78.

From expressions (10), (14) and (16) we have

$$\delta=D^{+}-N^{+}=\|A^{-}|^{+}_{4PQp_g}|^{+}_{Q}+N^{+}\cdot h-\||A^{-}|^{+}_{4PQp_g}|^{+}_{P}+ \\ +\frac{P}{2}\Big|^{+}_{P}+\frac{P}{2}. \tag{19}$$

From the latter/last expression it follows that

$$\min\delta>-1 \text{ and } \max\delta\leqslant h+1,$$

i.e. $0\leqslant\delta\leqslant h+1$.

Thus, for the calculation δ the value of further basis/base p_g must satisfy the following inequality:

$$p_g>h+2.$$

Let us consider the possibilities of using the inaccurate value of the rank of a number with the execution of rounding by the described method.

Let ranks ν_{C^+} and ν_M numbers $C^+=\left|\left|\left|A^-\right|^+_{4PQp_g}\right|^+_{p_g}+\frac{P}{2}\right|^+_P$ and $M=\left|\left|A^-\right|^+_{4PQp_g}\right|^+_Q$ be computed inaccurately, i.e.,

$$\nu_{C^+}=\Delta_{C^+}+\eta_{C^+}$$

and

$$\nu_M=\Delta_M+\eta_M,$$

where Δ_{C^+} and Δ_M - inaccurate ranks of numbers C^+ and M, and η_{C^+} and η_M - positive integer numbers, which show respectively, how ν_{C^+} and ν_M is more than Δ_{C^+} and Δ_M.

Let $\eta_{C^+}, \eta_M=0,1$, then from expression (19) follow that

$$\delta=\frac{M+N^+\cdot h+\frac{P}{2}-C^+-\eta_{C^+}\cdot P+\eta_M\cdot Q}{P}. \qquad (20)$$

From (20) we have

$$\min\delta>-2 \text{ and } \max\delta<h+3,$$

i.e.

$$-1<\delta<h+2.$$

Page 79.

Since from a technical point of view the correction of the result of rounding it is desirable to implement either only with the help of the addition or only with the help of the subtraction, we convert expression (20):

$$\delta'=\delta+1=\frac{\frac{3}{2}P+M+N^{+}\cdot h-C^{+}-\eta_{C^{+}}\cdot P+\eta_{M}\cdot Q}{P}.$$

We will obtain that the value of the correction δ' of the result of rounding satisfies the following inequalities:

$$\delta'\geqslant 0 \text{ and } \delta'\leqslant h+3.$$

consequently, further basis/base p_s it must answer condition $p_s\geqslant h+4$.

Thus, it is shown that for executing the operation of the rounding of relative numbers in SOK has the capability to use inaccurate values of ranks.

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Page 80.

SEARCH FOR THE OPTIMUM LAYOUT OF THE COMPUTER CENTERS IN A REPUBLIC NETWORK/GRID OF KAZAKH SSR.

V. Z. Brodskaya, T. V. Yevstifeyeva, I. G. Pil'shchikova.

During the creation of republic network/grid VTs first of all it is necessary to solve a question about the production/consumption/generation of its structure taking into account to the staging character of creation and series/row of the superimposed limitations.

Each region of republic has the central assembly point of information which in the preliminary stages either is had available the computational power (its own VTs), or it is attached to VTs of the nearest of the regions. The complex of the computer centers of the first stage possesses connections "each to each". A number of the computer centers in all stages is preset.

Initially stated problem of the synthesis of the network/grid of the computer centers was solved without taking into account the already available resources/lifetimes of computational power and lines of communications. In this case it was assumed that the information flows between all regions were known, the costs/values of transmission and information processing known and proportional to flow value, the capacities of the lines of communications and power of the computer centers were not limited and they completely correspond to load.

Upon this setting the problem sufficiently simply is solved with the help of heuristic algorithm [1], that also was done. The realization of this algorithm gave the even distribution of the computer centers according to the territory of republic with the small deviations to the side of the regions, which possess powerful/thick informational flows.

Then to the system were superimposed the limitations, which consider technical and some economic factors. The power of the computer centers were limited to the specific quantity of computers which it was proposed to establish/install on each of the regional centers.

Page 81.

Are introduced the account of the already existing lines of communications and the priority of the organization of some centers, which is caused by the already existing computational power in these points/items.

The superimposed limitations led to the need for introducing weight coefficients for all regions and matching system of penalties. If the volume of information due to the maintenance/servicing of adjacent regions exceeds the preset power VTs, each excess arbitrary unit of processed information is fined. With a small number of the computer centers in the network/grid (first stage) the further volumes of information to the low-power centers can serve as a reason for considerable above-plan expenditures. Were introduced penalties, also, for the cases, which require the laying of the new or further lines of communications.

Mathematical model in this case appears as follows. It is assumed that there is N of the assembly points of information, into each of which enters all information from the appropriate region. Let us designate each such point/item x_i

$$x_i \in X; \quad X = \{x_1, x_2, \ldots, x_N\},$$

then appropriate information P_i can be divided into two categories: the information which must be transmitted for the processing into other regions - $\overline{P^{ij}}$, and the information which can be directly processed - P^i. Thus, $P_i = P^i + \sum_j \overline{P^{ij}}$.

A number of points/items m, in which are placed VTs, varies from one to N_1 during the first stage and contains N of points/items in the final version of network/grid ($N_1<N$).

Penalty functions were accepted linearly depending on the volume of excess information. This is completely admissible, . . a number of laid channels, obviously, is proportional to the volume of the transmitted information, and a quantity of auxiliary equipment on VTs is proportional to the volume of the further information, processed by it. It is easy to show that factor of proportionality - constant value which is determined by the relation

$$A_{ij}=\frac{r_{ij}\cdot q_{ij}+Q}{r_{ij}\cdot q_{ij}},$$

where q_{ij} - channel capacity, and r_{ij} - cost/value of the transmission of informational unit along this channel.

Page 82.

Then the cost/value of the transmission of entire information in the direction of connection $x_ix_j - r_{ij}\cdot q\cdot n$, where n - certain integral coefficient, which is determining the capacity of information flow in direction x_ix_j. Penalty for the packing of these channels is defined as Q•n. It is obvious that with Q=0 $A_{ij}=1$.

During the limitations indicated the solution of the problem of the search for the optimum location of the preset quantity of the computer centers is reduced to the determination of the minimum of functional $F(Y_1, Y_2, \ldots, Y_m, X_1, X_2, \ldots, X_m)$, where $Y_1, Y_2, \ldots, Y_m$ - those points/items in which can be placed the computer centers, and $X_1, X_2, \ldots, X_m$ - corresponding to them subsets of the serviced points/items, which satisfy the relationships/ratios

$$Y_i \in X_i \subset X;\quad X_i \cap X_j = \varnothing\ (i \neq j);\quad Y \subset X.$$

In accordance with that presented the corrected heuristic algorithm of the search for the layout of the computer centers assumes preset set X and Y, the volumes of information which are exchanged any pairs $x_i x_j$ from X, cost/value of the transmission of informational unit between each pair of assemblies, cost/value of the transmission of the entire proceeding information from each element/cell x_i from the X to each element Y_k from Y, cost/value of processing informational unit on each VTs, the functional dependences, which are determining the cost/value of entire network/grid taking into account or without taking into account the limitations of any character. In the course of solution of task are determined or can be preset previously for each element/cell from Y its regions of possible replacement, which contain the enumerations of the adjacent assemblies or assemblies which can on one or the other reason replace data.

For the solution of the problems of the synthesis of the network/grid of the computer centers was written the program with the interchangeable units of the estimate of the cost of network/grid. Program is written in the language ALGOL-60 for the machine BESM-3M. Time of program translation from the algorithmic language to the language of loading 12 minutes. Program uses only working storage of machine. The scope of the program both the first and second version is approximately 1000 instructions.

The obtained results give the possibility to consider the most convenient versions of the organization of network/grid in the initial stage both in the cost sense and in the relation to the distribution of information in the system during the preset limitations.

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Page 83.

Organization of experiment during the research on the statistical models of exchange systems by information.

Ye. I. Domanovskaya, Ye. A. Pil'shchikov.

The tasks of the optimization of the systems of processing information (SOI) form the multidimensional space, constructed with many objective functions on many arguments: $Q=\{q_i(X)\}$, where $X=\{x_j\}, j=1, 2, \ldots, k_i$; Q - many tasks, X - many arguments, x_j — j- argument, q_i — i- objective function. Depending on the target of optimization the objective functions can come forward in the role of limitations.

As shown in work [1], the diversion of the tasks of optimization of SOI is usually great and their analytical solution in the contemporary stage is impossible. These and some other circumstances make it necessary to use methods stochastic simulation with the use/application of planning experiments for the reduction of the expenditures of time and resources for designing and research.

In this work is launched the attempt to consider the task of optimization of SOI as the task of the search for the global extremum of objective function in the space of large dimension. For shortening of capacity and time of simulation by the minimization of a number of accesses to model for calculating the objective function is proposed the algorithm, which varies the principle of search depending on situation and the combining global search of large step/pitch and adaptation without the immediate reaction with the local search of the slipping interval.

During the optimization of SCI of the preset structure on the criterion of the minimum of the time of the sequence of communications/reports on the network/grid we have

$$q_T = F(X_T, A_T, D_T),$$

where q_T - objective function; $X_T = (x_1, x_2, \ldots, x_n)$ - vector of the controlled parameters; $A_T = (a_1, a_2, \ldots, a_m)$ - vector of the uncontrolled parameters (vector of the environment of object); $D_T = (d_1, d_2, \ldots, d_p)$ - vector of the parameters, unattainable to measurement.

Page 84.

In our concrete/specific/actual version

$$X_T = (x_1, x_2, \ldots, x_7),$$

where x_1 - capacity ZU; x_2 - law of time allocation of delay in the channel; x_3 - quantity of access ducts; x_4 - discipline of servicing at the input of assembly; x_5 - discipline in the maintenance/servicing at the output/yield of assembly; x_6 - principle of the transmission of communications/reports; x_7 - algorithm of the selection of path.

$$A_T=(a_1, \ldots, a_6),$$

where a_1 - law of the distribution of the intensity of output flows; a_2 - quantity of categories of urgency; a_3 - law of the distribution of the lengths of communications/reports for each categories a_4 - matrix/die of flows on the network/grid; a_5 - matrix/die of the predicted lengths of the lines of communications between any pair of assemblies; a_6 - cost/value of the transmission of informational unit per the unit of the length of channel.

$$D_T=0,$$

$$B_T=(\{b_1^r\},\ b_2,\ \{b_3^r\}),$$

where b_1^r - time of the delivery/procurement of communications/reports to the r category; b_2 - expenditure for the creation of network/grid; b_3^r - probability of the delivery/procurement of reports of the r category.

Limitations they are respectively

$$b_1^r \leqslant \|t^r_{ja}\|,\ r=\overline{1,k};\ b_2<b_{2\max};\ b_3^r \leqslant \|b^r_{3Ta}\|.$$

The objective function

$$q_i = \sum_{r=1}^{k} c^r b_i^r;\ c^r = \text{const};$$

c^r - the weight of communications/reports to the r category.
Vector B is determined on stochastic model.

Page 85.

Arguments x_4, x_5, x_6 and x_7, which do not have direct quantitative characteristics and called arguments of the II kind, generate the multi-extremality by objective function SOI. Therefore it is necessary to solve the problem of the search for global extremum (G3), what is the most complicated search procedure of statistical character.

At large distances of target ($\rho > 7$) and number of degrees of freedom $n > 5$ the relation of losses to the search for the methods of the random search and method of gradient comprises

$$k \approx \frac{1}{\sqrt{n}}.$$

Therefore for the search G3 is expedient to use the algorithm of the random search of adaptation and inertness, called algorithm with director cone.

During the optimization of system ρ continuously it is reduced, in connection with which for an increase in the high speed and reliability it is proposed of the region suspected G3 to change the principle of search and to use a quasi-regular method with the slipping step/pitch (variety of the method of a variation in the metric).

Motion in the direction G3 is realized due to the reconstruction of the probabilistic characteristics of search, but being changed before to the new direction, system can make several steps in old direction and thus to surmount the "ridges/spines" of objective functions, which gives to method global character. Random tests/samples $(E_1, E_2, \ldots, E_m)$ are conducted in the preferable sector of directions with angle ψ at apex/vertex X, determined by the preset multivariate distribution (P(E/W), where $W=(w_1, w_2, \ldots, w_n)$ - parameter, which assigns mean direction, called the vector of experiment, E - vector of test step/pitch. the working step g is done in the direction of best test/sample $(X+gE^*_*)$. the sense of the vector of experiment W is determined by the "weighing" of the test directions

$$W=\frac{\sum_{i=1}^{m}\Delta q_{1i}E_i}{\left|\sum_{i=1}^{m}\Delta q_{1i}E_i\right|}.$$

The sign/criterion of the detection of region G3 and transition to LP is the motion of point X along the turns relative to certain center.

Thus, in proportion to the accumulation of information about the behavior of objective function the vector of storage W is turned/run up on the average in the direction, opposite gradient.

Page 86.

With the decrease of the expansion angle of cone ψ the possibilities of rotation W by one step/pitch are reduced, since is exhibited inertness. But with correct arrangement W (in the direction of "ravine") with decrease ψ are reduced losses to the search. And vice versa, with increase ψ system becomes more mobile due to an increase in the losses to the search.

Thus to the mobility and the losses to the search affect the parameters of search - angle of cone ψ, the length of the working step/pitch a, a number of tests/samples m. The density of distribution of the angle of working step/pitch (or path curvature), the characteristic mobility of search, is considered [2]:

$$P_m(\Omega) = m\frac{\left(\int\limits_{\psi}^{\Omega}\sin^{n-2}\varphi d\varphi\right)^{m-1}}{\left(\int\limits_{0}^{\psi}\sin^{n-2}\varphi d\varphi\right)^{m}}\sin^{n-2}\Omega;\ \text{with}\ n=m=2,$$

$$\bar{\Omega}_{cp}=\frac{\psi}{3}.$$

Losses to the search are calculated from the formula:

$$K=\frac{m}{\int_0^{\psi}\cos\Omega\cdot p(\Omega)d\Omega},\ \text{при}^{(1)}\ m=n=2,\ k=\frac{\psi^2}{2\sin^2\frac{\psi}{2}}.$$

Key: (1). with.

With increase ψ in the loss to the search they grow comparatively slowly.

One of the problems with the realization of search G3 is the fulfillment of the self-adjusting of algorithm - selection those optimum ψ, m, ensuring the retention/preservation/maintaining the necessary inertness and low losses.

For $q_i=F(X_T, A_T)$ as the initial parameters can be accepted in the absence of a priori information the following values:

1) Number of tests/samples on each step/pitch m=3;

2) the initial vector of experiment W_0, which has uniform density distribution of probability in the hypersphere of the parameters and which is determining on the first step/pitch on the random-number generator;

3) the expansion angle of cone $\psi=\frac{\pi}{4}$;

4) Initial coordinates of cone, which coincide since the origin of the coordinates;

5) the test step/pitch of search g, component along each coordinate $\frac{\Delta x_j}{10}$, where Δx_j - range of a change in the argument;

6) the working step/pitch a=g.

Page 87.

For the analysis of the suspected region G3 is proposed to use several modified algorithms of optimization with a variation in matrix, based on quadratic approximation of objective function with the use/application of slipping interval [3].

Since for determining q_i contemporary SOI is necessary the considerable operating time of computers, one of the important requirements to the algorithm of optimization is the minimization of a number of accesses for calculation q_i. The gradient method, used in work [3], requires for each point 2n of accesses to q_i (n - number of arguments); therefore for determination $\vec{S}_n$ it is proposed to use an algorithm of the static gradient, which makes it possible to find the evaluation of gradient g_n with number of tests/samples m<n.

Furthermore, for maximum use already obtained during the search G3 of information it is proposed as the direction on the first cycle in the local search $\vec{S}_l$ to use the sense of the vector of storage W at the latter/last step/pitch and to design therefore changes in direction $\Delta\vec{S}_n$ for the subsequent cycles.

Thus, in the algorithm LP is proposed this sequence of operations.

1. Evaluation of gradient

$$\vec{g}=\sum_{\eta=1}^{m}E_\eta[q_\gamma(X+\lambda E_\eta)-q_\gamma(X)],$$

where E_η - unit random vectors, evenly distributed in all directions of parameter spaces; λ - values of test step/pitch; X - initial state of system, from which are produced random tests/samples $(\lambda E_1, \lambda E_2, \ldots, \lambda E_m)$.

2. Calculation of increases in gradient

$$\vec{y}^{n-1}=\vec{g}^n-\vec{g}^{n-1}.$$

Page 88.

3. Calculation of change in direction of motion on cycle (besides the first)

$$\Delta\vec{S}_i^n=-\sum_{j=1}^{k}\left(\frac{a_i^{n-1}a_j^{n-1}}{\sum_{i=1}^{k}a_i^{n-1}y_j^{n-1}}-\frac{z_i^{n-1}\cdot z_j^{n-1}}{\sum_{i=1}^{k}z_i^{n-1}\cdot y_i^{n-1}}\right)\cdot g_j^n,\ j=1,2,\ldots,k,$$

where

$$Z_i^{n-1}=h_i^{n-1}\cdot y_i^{n-1};\quad h_{i,j}^{n}=h_{i,j}^{n-1}+\Delta h$$

(σ^{n-1} is determined in pcint/item 6).

4. Determination of sense of the vector of motion on this cycle

$$\vec{S}_i^{\,n}=\vec{S}_i^{\,n-1}+\Delta\vec{S}_i^{\,n}.$$

5. Determination of minimum in direction $\vec{S}_n$

$$\Phi(\lambda)=\min q(\vec{x}^{\,n}+\lambda\vec{S}^{\,n});$$

a) calculation by derived

$$a=\left.\frac{\partial\Phi(\lambda)}{\partial\lambda}\right|_{\lambda=0}=(\vec{S}_n)^T\vec{g}^{\,n},$$

b) test stap/pitch $P_i=x_i^{\,n}+\gamma S_i^{\,n};\ i=\overline{1,k},$

c) the calculation cf the derivative $a_1=\left.\frac{\partial\Phi(\lambda)}{\partial\lambda}\right|_{\lambda=\gamma}$

$$a_1=\frac{q[\vec{x}^{\,n}-(\lambda+\delta)\vec{S}_n]-q[\vec{x}^{\,n}+(\gamma-\delta)\vec{S}_n]}{2\delta},$$

where δ - projection of the vector of test steps/pitches λ in direction $\vec{S}^{\,n}$

$$\delta=\frac{(\vec{S}_n)^T\vec{\lambda}}{\|S^n\|}=\frac{\sum_{i=1}^{k}S_i^{\,n}\lambda_i}{\sqrt{\sum_{i=1}^{k}(S_i^{\,n})^2}},$$

d) the analysis: as a result of test step/pitch γ sufficiently they approached the extremum cr they passed it, for this we consider:

if $a<10a_1$, then is conducted linear interpolation on λ

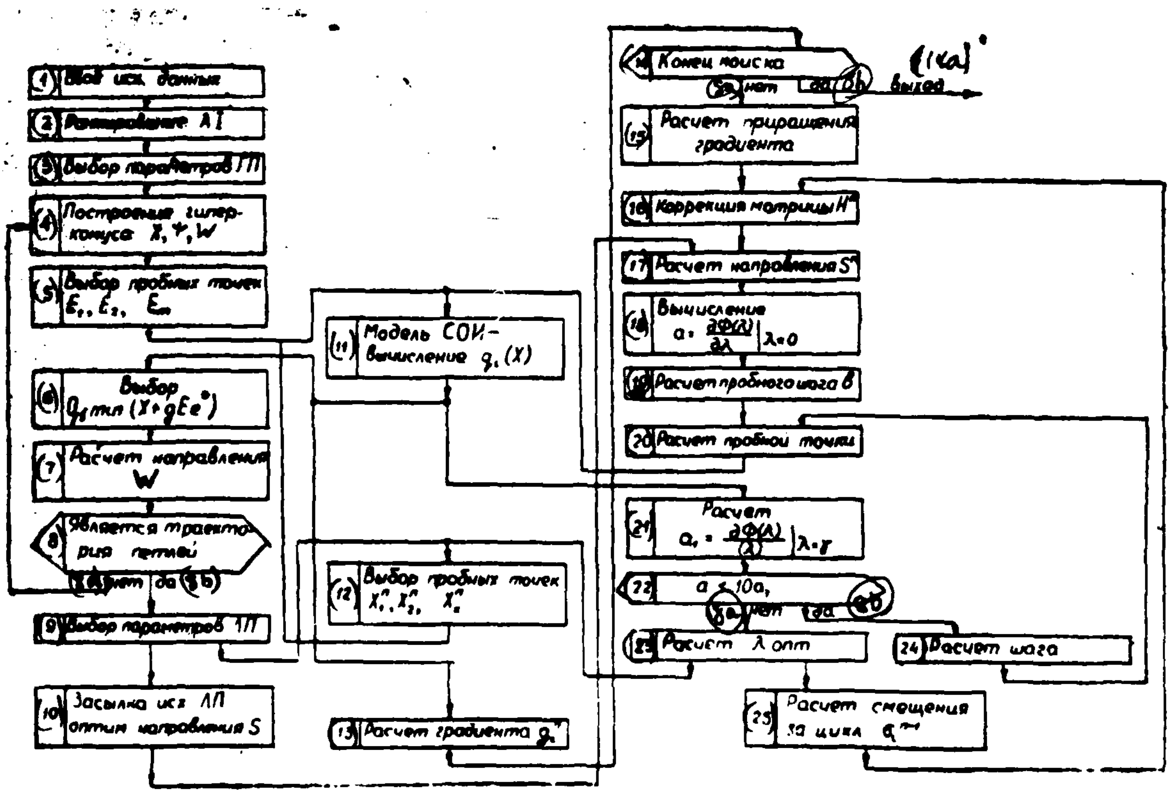
$$\lambda_{\text{опт}}=\frac{a\gamma}{a-a_1},\quad P_{\text{опт}}=x_i^{\,n}-\lambda_{\text{опт}}S_i^{\,n}.$$

If $\bar{a}>10\bar{a}_1$ (step/pitch is small), from point $\vec{P}(x)$ it is produced with the step/pitch $\bar{\gamma}=2\bar{\gamma}$ and $\bar{a}=\bar{a}_1$ new test step/pitch (transition to 5b) thus far we will not obtain extremum in direction S_i^n.

6. Determination of displacement of current point in cycle

$$a_i^{n-1}=P^n_{i\,\text{опт}}-x_i^n;\ i=1,2,\ldots,n.$$

Page 89.



Key: (1). Input of initial data. (2). Ranking. (3). Identification of parameters. (4). Construction of hypercone. (5). Selection of sampling points. (6). Selection. (7). Calculation of direction. (8). Is trajectory loop. (8a). even number. (8b). yes. (9). Identification of parameters. (10). Dispatching initial LP of optimum direction S. (11). Model SOI - calculation. (12). Selection of sampling points. (13). Calculation of gradient. (14). End of search. (14a). output/yield. (15). Calculation of increase in gradient. (16). Correction of matrix/die. (17). Calculation of direction. (18). Calculation. (19). Calculation of test step/pitch. (20). Calculation of sampling point. (21). Calculation. (22). nc key. (23). Calculation λ by wholesale. (24). Calculation of step/pitch. (25). Calculation of displacement in cycle.

Page 90.

Then we pass to point/item 1, beginning new cycle.

The operations of points/items 4-6 are presented in work [3].

The search for extremum is finished, when the modulus/module of gradient is reduced to the assigned magnitude

$$A=|\vec{g}|=\sqrt{\sum_{i=1}^{k} g_i^2}<\varepsilon.$$

The block diagram of algorithm is given on figure.

It should be noted that the use of this method in the examination of concrete/specific/actual systems assumes the possibility of the ranking of arguments of the II kind relative to one or the other objective function on the basis of the a priori information, which considers the specific character of particular task. If information about the effect of such arguments on q is insufficient for the ranking, then is accepted the uniform probability distribution of the selection of one or the other state of argument.

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Page 91.

Algorithm of the numbering of disorders.

V. N. Zaboltskiy, V. F. Lukin.

One of the directions of combinatory analysis is the solution of problems to numbering i.e., the establishment of a quantity of methods of executing some accurately described operations. In this case vital importance has the problem of identification which is reduced to the numbering of the elements of set of all methods of executing the preset operation. Are known the methods of the numbering of combinations [1] and transfers [2]. This article gives the method of the numbering of transfers $\langle a_1, a_2, \ldots, a_t \rangle$, formed from the elements/cells of series/row 1, 2, ..., t of which none occupies its natural place, i.e., $a_i \neq i (i=1, (1), t)$. Such transfers in work [3] are named disorders.

There are many E_t t-disorders $a_k = \langle a_1, a_2, \ldots a_t \rangle$ with a power of D_t.

It is known that

$$D_t = t!\left(1 + \sum_{i=1}^{t} (-1)^i \frac{1}{i!}\right)$$

or in the recurrent form

$$D_t = (t-1)(D_{t-1} + D_{t-2}). \qquad (1)$$

For disorder $a = \langle b_1, b_2, \ldots, b_t \rangle \in E_t$ it is necessary to determine number $0 \leqslant N_t \leqslant D_t - 1$ depending on the value of elements/cells b_i. For this purpose all disorders $a_t \in E_t$ somehow are ordered.

For the numbering of disorders is used expression (1). It is obtained on the basis of that fact that many E_t t-disorders can be decomposed into t-1 subsets

$$E_t = \bigcup_{r=1}^{t-1} E_{t_1}^{(r)}; \quad (E_{t_1}^{(r)} \bigcap_{\substack{r,s=1 \\ r \neq s}}^{t-1} E_{t_1}^{(s)} = \varnothing).$$

Page 92.

Each subset $E_{t_1}^{(r)}$ is characterized by the fact that do all have $a_t \in E_{t_1}^{(r)}$ $a_1 = \text{const}$. This makes it possible to consider $E_{t_1}^{(r)}$ as many disorders $a_{t_1}^{(r)}$ with a length of $t_1 < t$. $E_{t_1} \subset E_t$ - many t-disorders which have $a_1 = b_1$; $n(a_i)$ - number of place, which occupies element/cell a_i in the disorder.

Set E_{t_1} is divided/marked off into two subsets

$$E_{t_1} = E_{t_1^{(1)}} \cup E_{t_1^{(2)}} \text{ and } (E_{t_1^{(1)}} \cap E_{t_1^{(2)}} = \varnothing).$$

where $E_{t_1}^{(1)}$ - subset of t-2- disorders $a_{t_1}^{(1)}$, in which $n(1) = b_1$; $E_{t_1}^{(2)}$ - subset of t-1- disorders $a_{t_1}^{(2)}$, in which $n(1) \neq b_1$.

Taking into account the special feature/peculiarity of disorders $a_1 \neq 1$ and ordering disorders in set E_{t_1} in such a way that $N_{\alpha^{(1)}_{t_1}} < N_{\alpha^{(2)}_{t_1}}$, the number of this discrder α can be determined as follows:

$$N_\alpha = (b_1-2)(D_{t-1}+D_{t-2})+\varepsilon_1(D_{t-2}+N_{\alpha^{(2)}_{t_1}})+(1-\varepsilon_1)N_{\alpha^{(1)}_{t_1}}; \quad (2)$$

where

$$\varepsilon_1 = \begin{cases} 0 \text{ при } n(1)=b_1 \\ 1 \text{ при } n(1)\neq b_1 \end{cases};$$

Key: (1). with.

$N_{\alpha^{(p)}_{t_1}}$ - number of disorder α in set $E_{t_1^{(p)}}$ $(p=1, 2)$.

For determination $N_{\alpha^{(p)}_{t_1}}$ it is necessary tc lead disorder $\alpha^{(1)}_{t_1}$ either $\alpha^{(2)}_{t_1}$ into the confcrmity with the definition, to exclude element/cell b_1 and supplementarily in disorder $\alpha^{(1)}_{t_1}$ 1, to change the remaining elements/cells so that the disorder would be comprised of series/row 1, 2, ..., t-2 or 1, 2, ..., t-1.

In disorder $\alpha^{(1)}_{t_1}$ with a length of $t_1^{(1)}=t-2$ the elements/cells (besides b_1 and 1) are changed in the following form:

$$b_{i^{(1)}} = \begin{cases} b_{i+1+i_1^{(1)}}-1 \text{ при } b_{i+1+i_1^{(1)}}<b_1 \\ b_{i+1+i_1^{(1)}}-2 \text{ при } b_{i+1+i_1^{(1)}}>b_1 \end{cases}. \quad (3)$$

Key: (1). with

where

$$\delta_i^{(1)} = \begin{cases} 0 \text{ при } n(b_{i+1}) < n(1) \\ 1 \text{ при } n(b_{i+1}) > n(1), \end{cases}$$
$$i = 1, (1), t.$$

Key: (1). with.

Page 93.

For disorder $\alpha_{t_1}^{(2)}$ by length $t_1^{(2)} = t-1$ of a change in the elements/cells they are carried out through the following rule:

$$b_i^{(1)} = \begin{cases} b_{i+1} - 1 \text{ при } b_{i+1} \neq 1 \\ b_1 - 1 \text{ при } b_{i+1} = 1 \end{cases} \quad (4)$$

Key: (1). when.

On this is finished the procedure of numbering of this disorder taking into account element/cell b_1. For the total determination of its number it is necessary to repeat this procedure before obtaining of disorder with a length of $t_j = 3$ or $t_j = 2$. There is only two disorders with a length of $t_j = 3$ (their number N(2, 3, 1)=0, N(3, 1, 2)=1 and one disorder with a length of $t_j = 2$ (with number N(2, 1)=0).

General/common/total expression of the number of the disorder

$$N_a = \sum_j (b_1^{(j)} - 2)(D_{t_j-1} + D_{t_j-2}) + \varepsilon_j D_{t_j-2}, \quad (5)$$

where

$$\varepsilon_j = \begin{cases} 0 \text{ при } n(1) = b_1^{(j)} \\ 1 \text{ при } n(1) \neq b_1^{(j)} \end{cases}.$$

$$j = 0, 1, \ldots;\ b_1^{(0)} = b_1,\ t_0 = t.$$

Key: (1). with.

It is possible to show that the method of numbering proposed ensures mutual one-to-one correspondence between many t-disorders with a power of D_t and by the set of integers of the range $[0, D_t - 1]$.

Example. To determine the number of disorder α=<5, 4, 6, 2, 3, 7, 1>.

Using relationships/ratics (2), (3) and (4) we will obtain

$$\alpha = \langle 5,4,6,2,3,7,1 \rangle,\ N_\alpha = (5-2)(D_6 + D_5) + D_5 + N_{\alpha_{t_1}} =$$
$$= (5-2)(265+44) + 44 + N_{\alpha_{t_1}} = 971 + N_{\alpha_{t_1}};$$
$$\alpha_{t_1} = \langle 3,5,1,2,6,4 \rangle,\ N_{\alpha_{t_1}} = (3-2)(D_5 + D_4) + N_{\alpha_{t_2}} =$$
$$= (3-2)(44+9) + N_{\alpha_{t_2}} = 53 + N_{\alpha_{t_2}};$$
$$\alpha_{t_2} = \langle 3,1,4,2 \rangle,\ N_{\alpha_{t_2}} = (3-2)(D_3 + D_2) + D_2 + N_{\alpha_{t_3}} =$$
$$= (3-2)(2+1) + 1 + N_{\alpha_{t_3}} = 4 + N_{\alpha_{t_3}};$$
$$\alpha_{t_3} = \langle 2,3,1 \rangle,\ N_{\alpha_{t_3}} = 0;\ N_\alpha = 1028.$$

Page 94.

Let us consider reverse process - the determination of the value of elements/cells b_i of disorder $\alpha = \langle b_1, b_2, \ldots, b_t \rangle$ from its number. For the uniqueness of the reduction process it is necessary to indicate the length of disorder. The methodolcgy of the determination of the elements/cells of discrder α consists cf the consecutive determination of elements/cells $\overline{b_1^{(j)}}$ - the first elements/cells in the disorders with a length of $t_j \leqslant t$. beginning from element/cell $b_1^{(0)}$, and then in the consecutive transition from 2- or 3- disorder to the unknown t-disorder cn the basis of the relationships/ratios, "reverse" to relationships/ratics (3) and (4).

The first element/cell of the unknown disorder b_1 is determined by the solution of the inequality

$$N_\alpha \geqslant (b_1-2)(D_{t-1}+D_{t-2}). \qquad (6)$$

It is easy to show that length t_1 of disorder α_{t_1} can be found with research of the equality

$$\Delta_1 = N_\alpha - (b_1-2)(D_{t-1}+D_{t-2}) - D_{t-2}.$$

When $\Delta_1 \geqslant 0$ α_{t_1} it is a t-1- disorder, with ty a $\Delta_1<0$-t-2-disorder; n(1)=b_1. Further is determined number $N_{\alpha_{t_1}}$ of disorder α_{t_1}:

$$N_{\alpha_{t_1}} = N_\alpha - (b_1-2)(D_{t-1}+D_{t-2}) - \zeta_1 D_{t-2}. \qquad (7)$$

where

$$\zeta_1 = \begin{cases} 0 \text{ при}^{(1)}\ \Delta_1<0 \\ 1 \text{ при}^{(1)}\ \Delta_1 \geqslant 0. \end{cases}$$

Key: (1). with.

On this is finished the stage of the determination of the first element/cell b_1 of the unknown disorder α and the number $N_{\alpha_{t_1}}$ of disorder α_{t_1} with a length of $t_1 < t$. Carrying out a similar procedure, we determine the first elements/cells $b_1^{(j)}$ of t_j--disorders up to 2- or a 3- disorder. As a result it is obtained by one of three sequences: $<b_1, b_1^{(1)}, \ldots, b_1^{(t_f)}, 2, 1>$ either $<b_1, b_1^{(1)}, \ldots b_1^{(t_f)}, 2, 3, 1>$, or $<b_1, b_1^{(1)}, \ldots, b_1^{(t_f)}, 3, 1, 2>$. In each of these sequences the true element/cell of the unknown disorder is only b_1. All remaining elements/cells $b_1^{(j)}$ are true only for the disorders of corresponding length t_j. For determining all elements/cells of the unknown disorder consecutively/serially are restored on the known length and the first element/cell all disorders with a length of t_j, beginning with 4- or a 5-disorder.

Let be restored/reduced t_{j+1} disorder it is known that length t_j of disorder - $t_{j+1}+2$ (here j it makes the same sense, as in the expression (5)) and its first element/cell - $b_1^{(j)}$.

Page 95.

This means that in t_j- disorder $n^j(1) = b_1^{(j)}$, i.e. element/cell 1 occupies place with number $b_1^{(j)}$. Elements/cells t_j- disorder (besides $b_1^{(j)}$ and 1) are formed from elements/cells t_{j+1}- of disorder in the following manner:

$$b^{(j)}_{i+1+\gamma_i^{(j)}}=\begin{cases} b_i^{(j+1)}+2 & \text{при } b_i^{(j+1)}\geqslant b_1^{(j)}-1 \\ b_i^{(j+1)}+1 & \text{при } b_i^{(j+1)}<b_1^{(j)}-1 \end{cases}, \qquad (8)$$

Key: (1). with.

where

$$\gamma_i^{(j)}=\begin{cases} 0 & \text{при } n(b_i^{(j+1)})<n^{(j)}(1)-1 \\ 1 & \text{при } n(b_i^{(j+1)})\geqslant n^{(j)}(1)-1 \end{cases},$$

$$i=1, (1), t_{j+1}.$$

Key: (1). with.

If t_j- disorder has a length $t_{j+1}+1$, then its elements/cells (besides $b_1^{(j)}$) they are found from elements/cells t_{j+1}- disorder with the help of the relationship/ratio

$$b^{(j)}_{i+1}=\begin{cases} b_i^{(j+1)}+1 & \text{при } b_i^{(j+1)}\neq b_1^{(j)}-1 \\ 1 & \text{при } b_i^{(j+1)}=b_1^{(j)}-1 \end{cases}, \qquad (9)$$

$$i=1, (1), t_{j+1}.$$

Key: (1). with.

Example. To determine the elements/cells of disorder with a length of t=7, that has number N=1028.

Deciding inequality (6)

$$(b_1-2)(D_6+D_5)\leqslant 1028,$$

we find b_1=5.

After determining difference $\Delta_1>0$, we establish that $n(1)\neq b_1$ and

α_{t_1} is a 6-disorder. Its number

$$N_{\alpha_{t_1}}=N-(b_1-2)(D_6+D_5)-D_5=57.$$

First element/cell of discrder α_{t_1}

$$(b_1^{(1)}-2)(D_5+D_4)\leqslant 57,\ b_1^{(1)}=3.$$

Since $\Delta_2<0$, then $n^{(1)}(1)=b_1^{(1)}$ and α_{t_2} is a 4-disorder. Its number

$$N_{\alpha_{t_2}}=N_{\alpha_{t_1}}-(b_1^{(1)}-2)(D_5+D_4)=4.$$

The first element/cell of discrder α_{t_2}

$$(b_1^{(2)}-2)(D_3+D_2)\leqslant 4,\ b_1^{(2)}=3.$$

$\Delta_3=0$; therefore $n_{(1)}^{(2)}\neq b^{(2)}$ and α_{t_3} is a 3-disorder. Number of this disorder

$$N_{\alpha_{t_3}}=N_{\alpha_{t_2}}-(b_1^{(2)}-2)(D_3+D_2)-D_2=0.$$

Page 96.

This number has a 3-disorder $\alpha_{t_3} = \langle 2, 3, 1\rangle$.

Is restored from α_{t_3} with the help of expression (9) a 4-disorder α_{t_2}, which has $b_1^{(2)}=3-\alpha_{t_2}=\langle 3, 1, 4, 2\rangle$.

With the help of expression (8) is restored a 6-disorder α_{t_1}, which has $b_1^{(1)}=n_{(1)}^{(1)}=3$ $\alpha_{t_1}=\langle 3, 5, 1, 2, 6, 4\rangle$.

Similarly is restored the unknown disorder $\alpha=\langle 5, 4, 6, 2, 3, 7, 1\rangle$.

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Page 97.

The possibility of applying the code juice for the construction of combinatory switches.

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By the combinatory matrix switch (KMP) it is accepted to call the device/equipment, which consists of r of multicircuit transformers. The total number of windings on each transformer is not more than n+N, where n - number of input ones and N - number of output windings.

The windings of transformers, connected with n input KMP, form the input matrix/die of the connection

$$Q=[q_{il}]=\begin{bmatrix} q_{11} & q_{12} & \dots & q_{1n} \\ q_{21} & q_{22} & \dots & q_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ q_{r1} & q_{r2} & \dots & q_{rn} \end{bmatrix}.$$

Usually as matrix/die Q is used the code matrix/die of the n-bit positional code.

The windings of transformers, connected with N outputs/yields of KMP, form the output matrix/die of the connection

$$M=[m_{ij}]=\begin{bmatrix} m_{11} & m_{12} & \dots & m_{1N} \\ m_{21} & m_{22} & \dots & m_{2N} \\ \cdot & \cdot & \cdot & \cdot \\ m_{r1} & m_{r2} & \dots & m_{rN} \end{bmatrix}.$$

As matrix/die M can be used, for example, unit matrix of order $N \leqslant 2^n$.

The codes, used for the construction of KMP, can be considered as the class of the block binary surplus codes which are usually classed in the following manner:

C(n, d) - corrective codes with a minimum code distance of d;

D(n, d) - corrective codes with distance d_{ij}, limited from below and on top to one and the same value;

E(n, d) - corrective codes with the constant distance.

Page 98.

Value of the signal of the selected output/yield of KMP is always more than the signal of the unselected output/yield, and their relation in the worse case is determined by dependence [2]

$$C=\frac{n}{n-2d_{min}}.$$

The difficulty of the process of obtaining the corrective codes and the complexity of the devices/equipment of the conversion of the nonexcessive codes into those correcting restrain wide acceptance of KMP.

To the code of the system of residual classes (SOK), which possesses the excellent corrective properties, is at the same time distinctive the light and simple process of its obtaining.

This gives to us to right raise a question about the possibility of applying the code of SOK for the construction of KMP. For this purpose it is necessary to provide this code, which is nonpositional, the appropriate positional characteristics.

It is known that a number in the system of residual classes (SOK) can be unambiguously represented by the smallest non-negative deductions on the selected bases/bases $p_1, p_2, \ldots, p_n$, where $p_1, p_2, \ldots, p_n$ - mutually prime numbers, if number value does not exceed $\prod_{i=1}^{n} p_i$ [1].

Let at the system of bases/bases $p_1, p_2, \ldots, p_n, p_{n+1}$ of residual classes be is preset number $A=(\alpha_1, \alpha_2, \ldots, \alpha_n, \alpha_{n+1})$, lying in the range $P=\prod_{i=1}^{n} p_i$. In the theory of residual classes it is proved that the value of number A will not be changed, if we represent it in the system of bases/bases from which is withdrawn p_i (i.e. if we in representation

A delete digit a_i). 'Number A_i, obtained from A by the crossing out of digit a_i.' is called the projection of number A on basis/base p_i.

In view of the uniqueness of representation any two numbers A and B from range P must differ from each other in terms of deductions at least in one basis/base. If this is not implemented, then A and B are equal.

We will consider each of the bases/bases p_i certain i bit (i=1, 2, ..., n), the n-bit code, in which digit in the i bit is the least non-negative residue of a number on basis/base p_i. By analogy with the generalized theory of coding this code we will call p_i-th n-bit $P(p_i, n)$- code.

Page 99.

By analogy with the linear codes let us introduce for the code of SOK of the concept of the weight of code word and distance of Hemming (code distance) $P(p_i, n)$ code.

Definition 1. Weight w of code word $P(p_i, n)$- the code we will call the number of bases/bases (bits), according to which it has correct digits. (in our case of w=n).

Definition of 2. A number of positions (bases/bases, bits), in which two words $P(p_i, n)$-the code differ from each other in terms of their deductions, let us name Hemming's distance or simply by distance d, the small distance between two words on all pairs of code words $P(p_i, n)$-the code - minimum distance of code d_{min}.

Example 1. Let be preset basis $p_1=2$, $p_2=3$, $p_3=5$, $p_4=7$ system of residual classes. Range of the representation of numbers $P=p_1 \cdot p_2 \cdot p_3 \cdot p_4=210$.

Let us code on those accepted by the basis/base of two numbers A and B from range P.

Let A=159=(1, 0, 4, 5); B=201=(1, 0, 1, 5). All A is equal to weight B i.e. $w_A=w_B=4$. Given words differ in terms of deductions in basis/base p_3. Consequently, the code distance of them is equal to 1.

From the aforesaid it follows that if we on bases/bases p_1, p_2, ..., p_n form entire multitude of words $P=\prod_{i=1}^{n} p_i$, then the minimum distance between two words from range P on all pairs of words will be equal to unity.

Theorem 1. We have basis $p_1, p_2, \ldots, p_n, p_{n+1}$ of the system of residual classes, which satisfy condition $p_{n+1} > p_i$ $(i=1, 2, \ldots, n)$. If we on the bases/bases accepted form many words $P=\prod_{i=1}^{n} p_i$, then the minimum distance between all pairs of words will be equal to 2, i.e., $d_{min}=2$.

Proof. Let us take number $A=(\alpha_1, \alpha_2, \ldots, \alpha_n, \alpha_{n+1})$, lying/horizontal in the range P. If we in the representation of this number delete digit on any basis/base, then number value will not be changed, it will also belong range P. But numbers of this range have $d_P=1$. Consequently, the numbers, represented on n+1 to bases/bases, will have $d=d_P+1$. QED.

Corollary 1. let there be bases/bases p_1, p_2, ..., $p_n, p_{n+1}, \ldots, p_{n+k}$ the systems of residual classes, which satisfy condition

$$p_{n+k}>p_{n+k-1}>\ldots>p_{n+1}>p_i;\ (i=1, 2, \ldots, n),\ k=0, 1, 2, \ldots$$

Page 100.

If we on the bases/bases accepted form many words $P=\prod_{i=1}^{n} p_i$, then the minimum distance between all pairs of words will be equally k+1, i.e., $d_{min}=k+1$.

The proof of this corollary escape/ensues from the previous theorem.

We will call this code P(n, k)-code where n - number of working (informational) bits (bases/bases), k - number of control (redundant) bases/bases.

The introduced by us positional characteristics of the code in SOK such, as weight and minimum distance, make it possible to examine the procedure of the decoding of the code in SOK from the point of view of the linear codes.

For the linear positional systems is valid following theorem [3].

Theorem 2. In the simple systems of the type M with the symmetrical channels without the storage with any fixed/recorded procedure of coding the probability of the error of Rosh is minimum, if decoding is conducted through the criterion of the minimum of a number of noncoincident positions, i.e., if solution $\tilde{x}_m$ (transmitted communication/report x_m) is accepted every time that the combination $\tilde{Y}$, accepted differs from Y_m in a number of positions $d_{m'}$, less than from any other code combination:

$$d_{m'} = \min d_l,$$

or, which is the same,

$$d_{m'} < d_l, \text{ (для}^{(1)}\text{ всех } l).$$

Key: (1). for all 1.

In this case, if combination $\tilde{Y}_j$ differs in an identical number of positions from code combinations Y_m and Y_n, then with the equal success it can be accepted both the solution $\tilde{x}_m$, and solution $\tilde{x}_n$.

Let us recall that systems of the type M call the systems in which a number of possible solutions coincides with a number of possible communications/reports ($\tilde{M}$=M).

Let us attempt to establish/install, how should be carried out a procedure of coding so that it would make it possible to decode the signal accepted on the criterion of the maximum of likelihood ratio. The combinations of any code Y_k and Y_l (serrated signals Y_k and Y_l subset P) differ from each other in the specific number of positions d_{kl}.

Page 101.

Small of them they call Hemming's distance. From theorem 2 it follows that each subset $\tilde{Y}_m$ besides code combination $\tilde{Y}_m$ contains on at least all combinations, which differ from it in one, two and so forth t the positions:

$$t=\left[\frac{d-1}{2}\right],$$

where the sign [j] indicates whole part of d-1/2.

In other words, us interests such coding, with which the correct solution is accepted when after the demodulation of serrated signal it will seem that t (or less) of its elementary signals they were identified incorrectly. In this case we will indicate that selected code $P(p_i, n, k)$ corrects all errors by multiplicity t and less. On the basis of the aforesaid let us formulate theorem.

Theorem 3. So that the code would correct all errors on t and less to bases/bases, it is sufficient so that the code distance would be equally d=2t+1 or (that the same) so that the system would have k=2t surplus (control) bases/bases.

The proof of this theorem is obvious.

Let us demonstrate the following important theorem.

Theorem 4. Let be preset basis $p_1, p_2, \ldots, p_n, p_{n+1}, \ldots, p_{n+k}$ of the system of residual classes. In order to form many combinations $P=\prod_{i=1}^{n} p_i$ with d=k+1, it is sufficient so that would be satisfied the condition

$$p_i < p_{n+1} < p_{n+2} < \ldots < p_{n+k}, \quad i=1, 2, \ldots, n.$$

Proof. Let us demonstrate first theorem for n=1 and k=1. In this case, obviously, many combinations with d=2 will be equally p_1. Let $p_2 < p_1$. Then among many combinations p_1 there will be such, in which with different digits on basis/base p_1 of digit in basis/base p_2 they will coincide. It means, these combinations differ only in terms of digits of one basis/base p_1 and, therefore, them d=1, which contradicts condition. Then must be satisfied the condition of theorem $p_1<p_2$.

By mathematical induction thecrem can be proved, alsc, for n=2, by 3, ... and k=1, 2, 3:.

Previously we introduced the definition of the weight of code owl as the numbers of bases/bases according to which it has correct digits.

Page 102.

In the presence of n- working and k-control bases/bases the weight of a number is equal to n+k. Let us introduce now the concept of the weight of adjacent word, but let us before give its determination.

Let there be n working and k control bases of the system of the residual classes, which satisfy conditions of theorem 4. We form on the bases/bases accepted set $P=\prod_{i=1}^{n}p_i$ of code words. Let us register one of the words of this set $A=(\alpha_1, \alpha_2, \ldots, \alpha_n, \alpha_{n+1}, \ldots, \alpha_{n+k})$. According to theorem 4, the minimum number of bases/bases in terms of deductions by which differ all pairs of words from set P, is equal k+1. Consequently, among the combinations of set P are such, which coincide with the selected number A by deductions in n-1 to bases/bases.

We will call adjacent word the combination from set P, which has at least on one basis/base a deduction, which coincides with the deductions of the preset selected combination from set P.

Definition of 4. The weight of adjacent word w_c we will call maximum number of bases/bases in which the combinations from set P have identical deductions. If $P=\prod_{i=1}^{n} p_i$, then $w_c=n-1$. However, the weight of the selected code word P(n, k) of -code is equal to $w=w_c+d$, to the sum of the weight of adjacent word and code distance of this code.

It is here necessary to emphasize that the code distance d in P(n, k)-code increases only due to a quantity of introduced surplus bases/bases, but not due to their value. However, the value of them must satisfy the conditions of theorem 3.

In the code SOK an increase of the code distance is reached both due to the quantity and due to the value of surplus bases/bases. There is limit theorem [1]; according to it, the value of control basis/base is chosen on the basis of the condition for single-valued determination by the number of the interval into which falls the number, which contains error on one of the bases/bases, locations and values of error, i.e.

$$p_{n+1}>2p_n\cdot p_{n-1}.$$

Therefore is very important not to mix the concept of the code distance of the code into SOK with the concept of the code distance P(n, k) of -code.

Although these codes have one and the same origin, both they are residual. In the concept of redundancy in each of them is imbedded its individual sense. hence different will be the methods of their decoding, detection and correction of the errors in them.

Page 103.

On these qualities P(n, k)the -code is equivalent to the positional codes and, therefore, it can be used for the construction of combinatory switches.

Introduction P(n, k)-code substantially widens the field of application of the codes SOK and are offered the new possibilities before the system of residual classes which due to the obtained positional characteristics can, remaining nonpositional, to accept positional coloration.

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Page 104.

ORGANIZATION OF THE STRUCTURE OF THE SPECIAL-PURPOSE MAGNETIC DRUM.

V. P. Karyakin, P. D. Butakov, V. I. Shumay.

During the solution of the tasks, connected with the treatment/processing of the large files of information, it is necessary to preserve initial data and intermediate results not in OZU, but in the external or buffer storage TsVM. The transmission of information in OZU from VZU or BZU leads to the considerable time losses. External and buffer accumulators/storage are implemented most frequently on the magnetic data carriers. From these devices/equipment by maximum capacity/capacitance and the greatest access time possesses the accumulator/storage on the magnetic tape, by the minimum access time and by the smallest capacity/capacitance - drum store. The displacement/movement of information over the surface MB is realized both on time coordinates (direction of motion) and on the three-dimensional/space (lengthwise generatrix) due to the arrangement/position of heads, and on ML - only on time coordinates.

Let us consider the location of heads MB., which makes it possible to realize the algorithm of rapid Fourier transform (BPF). This conversion is realized by multichart use/application of the specific sequence of the uniform elementary conversions on numbers of initial file of information. Realization of BPF with the help of the magnetic drum can be presented as follows: read out with MB air of numbers enters AU it is finished by the moment/torque of approach under the heads of the following pair of numbers. During the following stroke/cycle of reading it occurs to wash down the obtained results.

For convenience in the following presentation let us introduce two determinations.

Definition 1. under distance of S between the pair of read (written/recorded) numbers we will understand a quantity of numbers n, registered with the drum between two numbers indicated, assuming that numbers are registered into one column.

Page 105.

Definition of 2. Column we will call the sequence of the numbers, registered in the circumference of drum.

The structure of arrangement/position on MB. Initial data, and also intermediate and final results depends on translation algorithm. For determining the requirements for the arrangement/position of information and heads of recording - reading on MB let us analyze algorithms BPF, using a representation of algorithms by the current graphs/counts of conversion [1].

In the graph/count with the recurrent structure (Fig. 1) the distances between the pairs of read and written/recorded numbers are equal for the single stages of conversion, and from one stage to the next changing is proportional of the degree of number 2, moreover in the stage they remain constants. The same property possess nonrecurrent graphs/counts (Fig. 2). The structure of regular graph/count is characterized by constancy of the distance between the read and written/recorded numbers in all stages (Fig. 3).

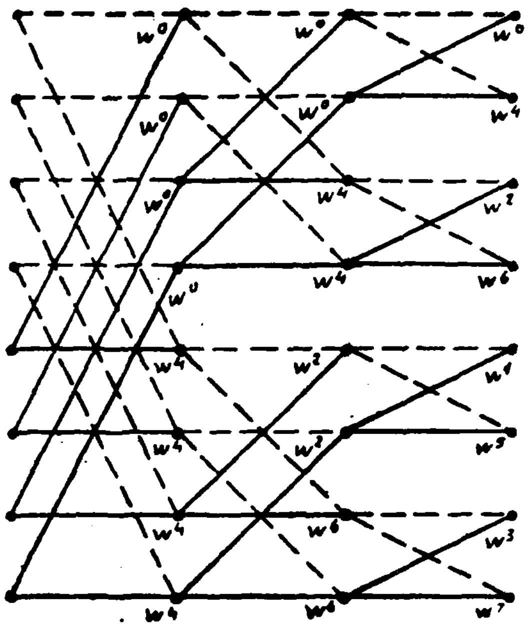

Fig. 1. Regular graph/count.

Page 106.

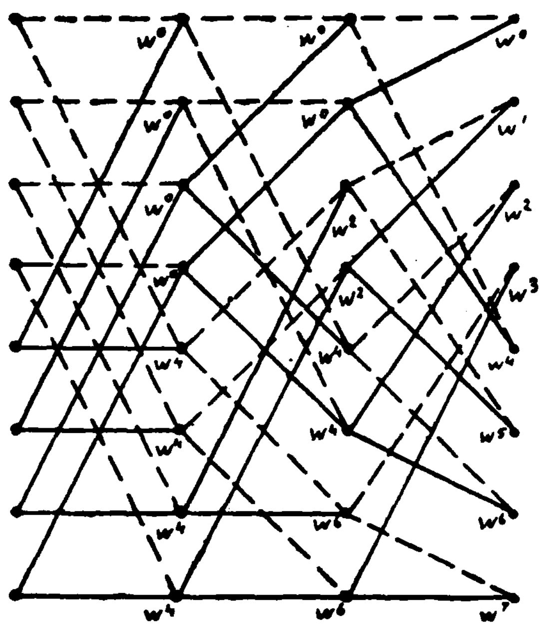

Fig. 2.

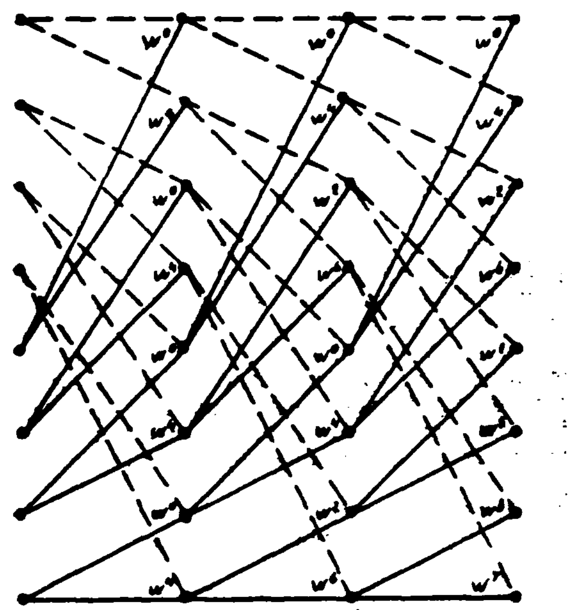

Fig. 3.

Fig. 2. Recurrent graph/ccunt.

Fig. 3. Nonrecurrent graph/count.

Page 107.

For the analysis of the versions of realization of BPF let us consider the simplified model of drum. Let us assume that for reading or recording of numbers is necessary only one head, in this case a number is read or is written/recorded by all digits simultaneously. Let us assume also that the processing, numbers occurs instantly, i.e., the counted number without the delay in AU enters the channel of the recording where it is written/recorded to the drum.

In general law of the arrangement/position of magnetic recording heads depends not only on the graph/count of conversion, but also on the method of positioning/arranging the numbers on MB, i.e., from a quantity of columns. Thus, during the arrangement/position of heads on MB it is necessary to consider three parameters - time of the start of head, numbers of columns, in which are registered the data, and the form of the graph/count of conversion.

Let us determine the law of the arrangement/position of heads, i.e., number and the order of their location above the surface MB for the regular structure of graph/count during the recording of numbers into one column. Since the regular structure is characterized by constancy between pairs of the read and written/recorded numbers in all stages, it suffices to determine the quantity of heads, necessary for conducting one stage. Let the file of initial data be registered to MB in the natural order and number of samples in it $N=2^n$. The distance between the read numbers will be equally N/2, and between those written/recorded - 1 (Fig. 4). during the gyration of drum will occur the "compression" of the written/recorded file. In the second stroke/cycle of recording the first recording head - ZG1 - will write/record a number to the previcusly registered zone. In order to avoid the imposition of recording, it is necessary to introduce further head ZG3 and to carry out the recording by heads ZG2 and ZG3. In the third also of recording it is necessary to include/connect one additional head of reccrding - ZG4 - to avoid the imposition of information, which can arise during the recording by head ZG2, and so forth. The total number of reccrding heads is equal (N/2+1), where N = length of array.

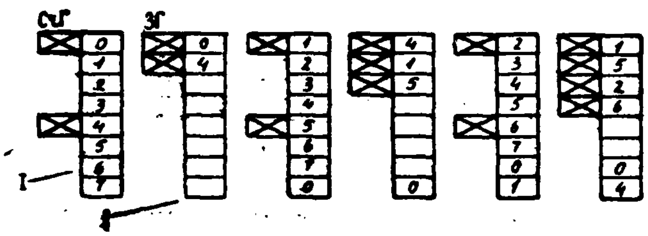

Fig. 4. Realization of regular structure during the recording of file intc cne cclumn.

Page 108.

If the file of initial data was registered in a binary-inverse order, then a number of recording heads is equal to 2, and reading - (N/2+1).

Let us consider the realization of the same graph/count during the recording of file intc twc columns, on N/2 samples in each. With this method of the recording the distance between the reading heads and their location from one stage tc the next will vary proportional the degree of number 2. Figure 5 shows two first strokes/cycles of the work of heads in three consecutive stages. Recording heads it is necessary only 2, and reading - 2(2n-1), where $n=\log_2 N$. It is necessary to note that cnly in first stage the reading is satisfied by the pair of heads frcm the different cclumns, in all remaining stages in the stroke/cycle of reading work the heads of one column.

If we "roll" file on the drum, i.e., we rewrite him of one zone into another and back, then the total quantity of heads comprises $2k_{max}+m$, where k_{max} - maximum number of heads in any stage, m - number of further heads, which permit implemention of a conversion in the remaining stages.

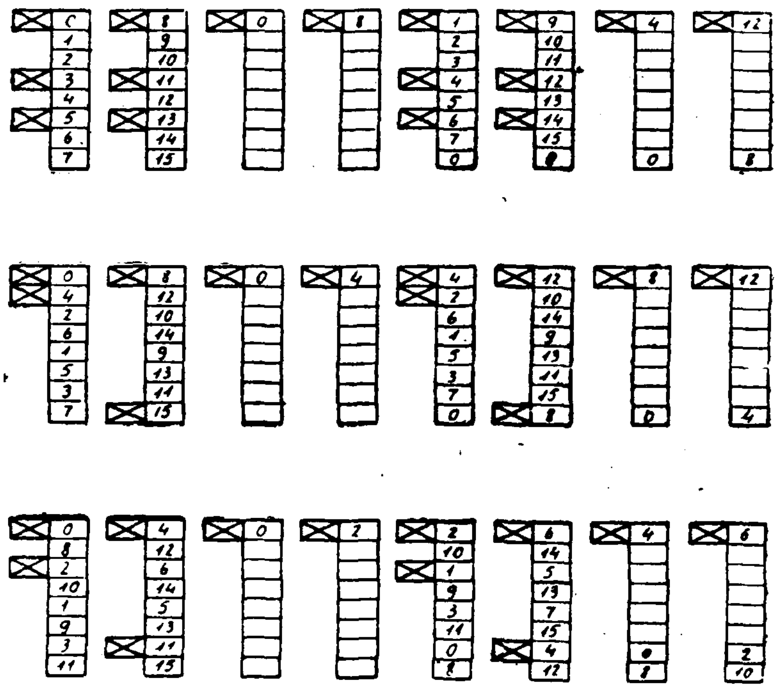

Fig. 5. Realization of regular structure during the recording of file into two columns.

Page 109.

For the described versions of the realization of regular graph/count this respectively is 2(N/2+1) heads in the first case and 2(n+2) - the secondly.

Let us consider graph/count with the recurrent structure and it is determined the place of the heads above the drum surface for the file, registered into one column. For this structure is characteristic the constancy of the distances between the pairs of read and written/recorded numbers "within" the single stage: in k stage $S_k=N/2^k$, where k=1, 2, ..., n. As can be seen from figure 6, during the first stage it suffices to have two reading and two write heads, in the second stage a number of heads grows to three; in the k stage it reaches $(2^{k-1}+1)$. For this graph/count the results can be written/recorded directly in the place of the read numbers, which makes it possible to lead all the conversion in one column.

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THEORY OF CODING INFORMATIONAL SIMULATION.(U)

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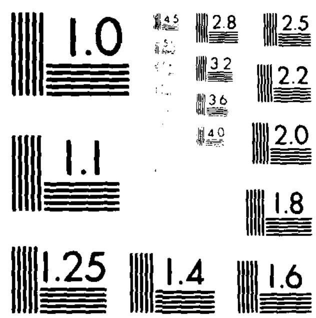
1.0
1.1
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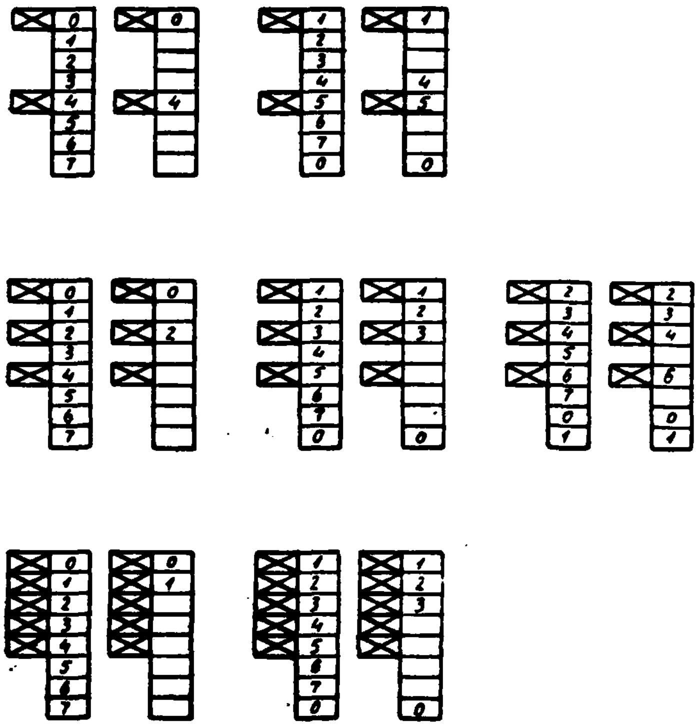

Fig. 6. Realization of recurrent structure.

Page 110.

The recording of the file of data into two columns gives the same location of heads, as in the case of regular graph/count. During the motion of file along the drum the total number of heads will comprise $\sum_{i=1}^{n} k_i$, where k_i - number of heads in the i stage.

For the nonrecurrent graphs/counts during the recording of the file of numbers into one column on each stage it is necessary to have two those recording and $(2^{k-1}+1)$ reading heads (k - number of stage). During the arrangement/position of the file of the numbers into two and more than columns the reading and recording can be carried out both of one and from several columns, in this case an optimum number of columns is connected with the basis/base of conversion. For basis/base p=2 an optimum number of columns j=2, for p=4 j=4, since this simplifies equipment.

A deficiency/lack in the regular and recurrent structure is obtaining results in binary- inversely the order during the location of initial file in the natural order, which leads to the need for classification. This operation can be realized in the order, reverse to initial conversion, which doubles temporary/time expenditures. Another method of classification consists in the setting up of further heads in the latter/last stage. The law of the location of these heads during the arrangement/position of file into two columns is described by the formula

$$S_k = S_i + \dot{r}, \qquad (1)$$

where $k = N/2^{r+2} - 2^r = 2^{n-(r+2)} - 2^r;$

$$k = 2^r + i;\ r = 1, 2, \ldots, (n-2);\ i = 1, 2, \ldots, 2^{r-1}.$$

S_k - distance between the head, which reads the k-th pair of numbers, and initial, zero, the head, which reads the first pair of numbers.

If as a result of calculations it is obtained, that $S_k = S_j$ with $k \neq j$, then this it means that the k-th and j-th pairs of numbers reads one head. If values S_k are negative, then head with number S_k is arranged/located relative to S_0 in side, opposite $S_k > 0$. Tables 1 and 2 give distances S_k for the reading heads, calculated by formula (1) for the files in length 128 and 256 numbers. For both files for the classification of the results it is necessary of 27 heads. Increase in the number of these heads to occur only with increase in $n = \log_2 N$ by 2. Therefore for decreasing the equipment the files of the data with a length of $N = 2^n$ with odd n must be transformed, leading them to the files with even n. This can be carried out, after introducing two channels of recording, i.e., after dividing file into two parts.

Page 111.

Table 1.

0	0	12	18	30	33	45	51	63	0
1	1	13	19	31					31
2	2	14			35	47			14
3	3	15							45
4	4		22		37		55		4
5	5		23						35
6	6				39				18
7	7								49
8	8		26		41		59		−4
9	9		27						27
10	10				43				10
11	11								41
12		16	28			49	61		−14
13		17	29						17
14		20				53			−10
15		21							21
16		24				57			−18
17		25							13
18				32	44	50	62		−31
19				34	46				−17
20				36		54			−27
21				38					−13
22				40		58			−35
23				42					−21
24					48		60		−45
25					52				−41
26					56				−49

Page 112.

Table 2.

0	0	8	20	28	34	42	54	62	65	73	85	93	99	107	119	127	0
1	1	9	21	29	35	43	55	63									63
2	2	10	22	30					67	75	87	95					30
3	3	11	23	31													93
4	4	12			38	46			69	77			103	111			12
5	5	13			39	47											75
6	6	14							71	79							42
7	7	15															105
8		16	24			50	58			81	89			115	123		−12
9		17	25			51	59										51
10		18	26							83	91						18
11		19	27														81
12				32	40	52	60					97	105	117	125		−30
13				33	41	53											33
14				36	44												−18
15				37	45					82	90						−45
16					48	56								113	121		−42
17					49	57											21
18							64		72	84	92	98	106	118	126		−63
19							66		74	86	94						−33
20							68		76			102	110				−51
21							70		78								−21
22									80	88			114	122			−75
23											96	104	116	124			−93
24											100	108					−81
25											101	109					−105
26												112	120				−105

Page 113.

Special interest is of the classification of initial file during the first stage when heads are located from each other at the distances, calculated according to formula (1). Beginning from the second stage, a number of reading heads is equal to 2 in each column, and the distances between them are changed from the stage to the stage proportionally of the degree of pair. A number of recording heads is equal to 2 in each stage, and their alignment is not changed. On MB with this structure of heads the results of conversion are obtained in the natural order, i.e., this arrangement/position of information and heads realizes graph/count with the nonrecurrent structure.

From the given analysis it follows that a minimum number of heads is obtained during the use of recurrent and nonrecurrent graphs/counts during the recording of files into two columns. These two structures can be virtually realized. In both cases the distance between the heads must be equal to one number, i.e., the length of a number on the drum determines the distance between the heads, which depends also on the geometric dimensions of head. Therefore the minimum distance between the heads is determined either by the length of the section of a surface of MB, the necessary for the recording number or, if the length of this section is lower than the technically attainable distance between the heads, the geometric dimensions of the heads of recording - reading. Since heads are placed on MB at the distances, proportional of the degree of pair, then the length of circumference of MB is determined by the length of the workable file that it is an essential deficiency/lack in the described structures. To eliminate the effect of the sizes/dimensions of magnetic recording heads on the minimum distance between them is possible by the introduction to the duplicated/backed up/reinforced recording. Then, as can be seen from figure 7, it is possible to spread by series/row the heads confronting into the different columns. In this case the minimum distance between them will be determined only by the length of the number, written/recorded on MB. By a basic difference in the drum in question from MB, it is used in TsVM, is transferring in one column. MB with a similar method of access already exist [2]. For the realization of conveyor data processing it is necessary to organize the motion of files along the drum, i.e., during the recording of the results of the first stage into second column into the first column it is necessary to write/record the new file of data, thereby realizing a continuous motion of files along the drum.

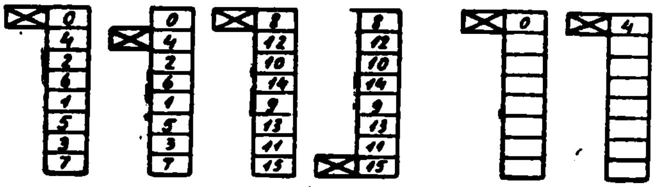

Fig. 7. Realization of nonrecurrent structure.

Page 114.

If during the "fluctation" of file the time of processing group of k files comprises T=knt, where k - number of files, t - time of transformation in one stage, $n=\log_2 N$, then during the motion of files it is aqual T=(k+n-1)t. It is possible to decrease the conversion time of the group of files, if we begin the operations of the following stage, without expecting the termination of previous. The conversion time of one file will compose $t(2+2(n-2)/N)$, where t - conversion time of one stage (time of the revolution of drum). For N=1024, t=10 ms, $T \cong 20$ ms.

Above was examined the ideal model of drum. In the implementation of the described structures one should consider that a number with MB is read consecutively/serially or is parallel-series, moreover proportionally to an increase in the paths/tracks grows equipment. It is necessary to consider that the results will be written/recorded at the moment of the reading of the following pair of numbers, i.e., the beginning of file in each stage will be displaced by one word. However, the technical realization of special difficulties does not represent.

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Page 115.

ONE TASK OF SCHEDULING.

E. N. Liang, V. A. Ustinov, N. A. Utembayev.

At present very important in applied sense is the task of scheduling theory, called more frequent the task of scheduling. Its overall diagram can be described as follows. There are by m of machine tools and n of the parts each of which must pass processing on all machine tools in the determined order. This order can be identical for all parts or different for their different groups. In this case the operations are considered indivisible (after beginning processing the i part on the j machine tool, we they must bring it to the end).

Is preset matrix/die $A=\|a_{ij}\|$, where $a_{ij}\geq 0$ - time, necessary for processing of part ($a_{ij}=0$, if the i part is not treated on the j machine tool). It is necessary to indicate this starting sequence of parts in the processing, which would minimize the total time of processing all parts, i.e., the length of productive cycle.

to the direct solution (via straight/direct sorting all versions) this task does not yield, since already in the simplest case of the identical passage of parts it would be necessary to choose smallest from n ! values. The formulation of the problems of this class in the form of the tasks of integral linear programming is one of the promising methods of resolution of problem.

Formulation of the problem. Let us consider general problem of scheduling in connection with ccrcrete/specific/actual object. As the subject of research is undertaken the production section "steel foundries - the isolation/evolution of soaking pits". Karaganda metallurgical combine.

Page 116.

Production process in this section carries discrete/digital character and represents the series/row of the consecutive operations of working metal (meltings or ingots):

1) the smelting of metal in the Martin and converter shops;

2) casting into the casting forms;

3) residue after casting (crystallization);

4) stripping the "undressing" of melting (blasting/detriment of ingots or the removal/taking casting forms) before its settlement into the heating nuclei ONK;

5) settlement and heating ingots in the nuclei ONK;

6) the rolling of ingots on the roughing mill.

The length of metal working on different operations is the random parameter, depending on the method of smelting metal (converter or Martin) and from the type of ingots.

In the technological cycle of metallurgical plant the section "steel foundries - ONK" is one of the most complicated ones. Nonuniform output of metal in the steel foundries leads, on one hand, to idle times of the roughing mill, and on the other hand - to the delay of output into the rolling of the heated metal, which causes an increase in the scale formation, the further consumption of fuel, etc.

Furthermore, for guaranteeing the rhythmic work of the roughing mill it is necessary to approach the decrease of a quantity of readjustments of mill, which sets the specific limitations by an order of the issue of meltings according to the types of ingots.

The task of the diurnal planning/gliding of the issue of meltings in the steel foundries is formulated in the terms of the task of scheduling whose setting is given in work [1]. The special feature/peculiarity, which differs the task in question from the general/common/total setting, is the introduction of further limitations to a length of repair and a quantity of overhauled nuclei ONK:

$$\bar{t}_{kl} - \underline{t}_{kl} \leq P, \qquad (1)$$

where $\bar{t}_{kl}$ - time of the termination of the repair (k - identification number, l - number of the group of aggregates/units);

$\underline{t}_{kl}$ - time of the beginning of repair;

P - standardized/normalized length of the repair (it depends on the type of aggregate/unit and means of repair).

$$\sum_{s=1} x_s z_s = \text{[illegible]} \qquad (2)$$

where $Z_s = \begin{cases} 1, \text{ если ячейка находится в ремонте; (1)} \\ 0, \text{ в противном случае. (2)} \end{cases}$

Key: (1). if nucleus is located in the repair; (2). otherwise.

Page 117.

Evaluation criteria cf the graph of the issue of meltings must consider requirement of the minimum of the length of production process, time, necessary for the readjustment of mill, and also losses during the distribution cf ingots according to the nuclei cf ONK. Thus, the criterion cf the quality of planning/gliding following:

$$F(p)=\sum_{i=1}^{k}a_i\cdot\tau_i+\sum_{j=1}^{m-1}\sum_{i=1}^{n}(t_{ij+1}-\bar{t}_{ij})\beta_{ij}+\Phi(t). \quad (3)$$

where τ_i - period of the readjustment of mill;

k - quantity of readjustments;

$(t_{ij+1}-\bar{t}_{ij})$ - latency of the i claim before maintenance/servicing (j+1) th by aggregate/unit;

m - quantity of aggregates/units of maintenance/servicing;

n - quantity of claims;

P(t) - certain function, which calculates the distribution of ingots on the basis of the nuclei ONK and the state cf nuclei.

The essential feature of task is the fact that the time of processing any i claim for the j aggregate/unit is random variable with known law of distribution. Let us introduce into the function of target (3) random vector c, into which in general enter the random noise and the control pressures. As a result the task of scheduling the optimum the issue of meltings for the 1 realization is reduced to the minimization of the functional

$$I_l(c) = \int_p F(p, c) \cdot f(p) \cdot dp. \qquad (4)$$

The quality of planning/gliding is considered on the statistical model of the section in question. Therefore finally we obtain

$$I^{**}(c) = \frac{\sum_{l=1}^{N} I_l^*}{N}, \qquad (5)$$

where N - quantity of realizations;

$I_l^* = \frac{1}{m}\sum_{\nu=1}^{m} I_\nu(c)$ - discrete/digital analog of the function of target (4);

m - many plans/layouts.

$$T^x_{ij} \leq t_{ij} + T_{ij}(x), \qquad (6)$$

$$t^x_{ij} \leq t^x_{ij} + 1, \qquad (7)$$

$$t_{sl} - t_{sl} < P, \qquad (8)$$

$$\sum_{s=1}^{m} R_s \cdot Z_s = 2, \qquad (9)$$

where (6) - continuity condition of operation, (7) - the condition of the impossibility of perfecting melting simultaneously on two aggregates/units, (8) - limitation in the length of repair, (9) - limitation in a quantity of overhauled nuclei in ONK.

Page 118.

Algorithm of the solution. The task in question relates to class of the combinatory tasks of discrete/digital programming. The multiple methods of their solution are heterogeneous by the character, but they all to a certain degree use an idea of sorting. By the most widely used method, which makes it possible to decrease a number after subduing, is the "method of branches and borders". Is close to it the "additive algorithm" of Balash and its modification, the "simplex method with the cessations" and so forth.

This problem it is proposed to solve the "method of branches and borders". Its basic idea consists of the consecutive step-by-step separation of many permissible solutions [2]. The evaluation of the solution at each step/pitch of separation is conducted on the statistical model of the section in question. At the basis of the separation of many permissible versions of the graph of the issue of meltings lie/rest the following considerations. Entire multitude of meltings can be decomposed into several groups according to the types of casting forms, let us assume to three: m, l, n. In turn, each of m, l, n of the numbers of meltings consists of two types of meltings - KP and SP. A quantity of permissible versions at the first step/pitch is equal to the number of permutation of three elements/cells. For each version on statistical model are computed the values of functional. From them is chosen minimum. At the following step/pitch many meltings, selected on the minimum of functional, are divided/marked off into three subsets, differing from each other by the fact that in one of the groups m, l, n is changed sequence of the types of meltings. Is computed the value of the functional of each subset. And so until is found the close to the optimum graph of the issue of meltings.

Some results. On the statistical model of the section of metallurgical plant was traced the functioning of converter shop for the purpose of the determination of the necessary quantity of pouring floors under the following conditions and during the limitations: work two converters by productivity to 25 meltings each with the average period of melting one hour, for the average delay time of composition after the casting of 0.5 hours, reliability 0.95; the smelted melting is immediately serviced/maintained by pouring floor.

Page 119.

In the formulated task they were used the method of branches and borders in the simplified setting, the obtained results showed the efficiency of algorithm and the adequacy of model object.

As a result of repeated playback on the computers BESM-3M were obtained the following results: for the satisfaction of the preset conditions and requirements is necessary the presence of six pouring floors; the mean delay time of the compositions before the stripping isolation/evolution due to the queue of 2.4 hours, the mean shutoff period of the stripping isolation/evolution of 15.9 hours.

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Page 120.

The numbering of the elements[redacted] of the subsets of the separation of apexes[redacted] a n- graduated binary Cuba, by the linear form realized.

L. N. Muk~~a~~yeva.

Let on many Σ_n apexes/vertexes of n-dimensional binary cube is determined the linear form

$$L_n(\bar{a}, \bar{x}) = \sum_{k=1}^{n} a_k x_k. \qquad (1)$$

Here vector $\bar{a}$ determines form factors; $\bar{x}$ - variable/alternating binary vector. When vector $\bar{x}$ passes set Σ_n, form accepts sequence G of the values:

$$G = \{r_1, r_2, \ldots, r_m\}, \ (m < 2^n).$$

In accordance with this form (1) realizes a separation of set Σ_n into the classes of equivalency σ_α:

$$\Sigma_n = \bigcup_{\alpha=1}^{m} \sigma_\alpha, \quad \sigma_\alpha \cap \sigma_\beta = \varnothing \ \overset{(1)}{\text{при}} \ \alpha \neq \beta.$$

Key: (1). with. where each subset σ_α is determined by the condition

$$\sigma_\alpha = \{\bar{x} / L_n(\bar{a}, \bar{x}) = r_\alpha\}.$$

In the article is placed the task: to indicate the principle of the single-valued numbering of the elements/cells of each of the subsets σ_s.

Without the limitation of generality it is possible to consider that all coefficients a_k of form (1) are different from zero and are arranged/located in the order

$$a_1 < a_2 < \ldots < a_n.$$

Us interests the case when $S < 2^n$ and a_k - integers. The case, when a_k are rational, easily is reduced to the case of linear form with the whole coefficients.

Page 121.

Algorithm of the determination of a number of apexes/vertexes of subset σ_s

Let us designate by symbol $S_n(\bar{a}, r_s)$ a number of apexes/vertexes of subset σ_s.

Assertion 1. the generating function of the sequence of numbers $S_n(\bar{a}, r_s)$, $(r_s \in G)$ takes the form:

$$F_n(x) = \prod_{s=1}^{n}(1 + x^{a_s}) = \sum_{k=\tau_1}^{\tau_2} S_n(\bar{a}, k)x^k, \qquad (2)$$

where $\tau_1 = \min_{\bar{x} \in I_n} \{L_n(\bar{a}, \bar{x})\}$,

$\tau_2 = \max_{\bar{x} \in I_n} \{L_n(\bar{a}, \bar{x})\}$.

Proof follows directly from the disclosure/expansion of product $\prod_{s=1}^{n}(1+x^{a_s})$.

Let us consider procedure of the calculation of values $S_n(\bar{a}, r_s)$.

If $r_1 \geqslant 0$, then function $F_n(x)$ let us leave without the change; if $r_1 < 0$, then instead of function $F_n(x)$ let us consider $\Phi_n(x)$:

$$\Phi_n(x) = x^{-r_1} \cdot F_n(x).$$

It is obvious,

$$\Phi_n(x) = \prod_{s=1}^{n}(1+x^{a'_s}),$$

where

$$a'_s = |a_s|.$$

Let

$$\Phi_n(x) = \sum_{j=0}^{r_2-r_1} S_n(\bar{a}, j+r_1)x^j.$$

Since

$$\frac{\Phi'_n(x)}{\Phi_n(x)} = \sum_{s=1}^{n} a_s \frac{x^{a_s-1}}{1+x^{a_s}} = \phi_n(x),$$

then

$$\Phi'_n(x) = \phi_n(x) \cdot \Phi_n(x).$$

Consequently,

$$\Phi_n^{(k+1)}(x) = \sum_{i=0}^{k}\binom{k}{i}\Phi_n^{(i)}(x) \cdot \phi_n^{(k-i)}(x).$$

Page 122.

Hence

$$(k+1)\frac{\Phi_n^{(k+1)}(0)}{(k+1)!}=\sum_{i=0}^{k}\frac{\Phi_n^{(i)}(0)}{i!}\frac{\psi_n^{(k-i)}(0)}{(k-i)!}. \qquad (3)$$

Let $\psi_n(x)$ be decomposed/expanded in the Maclaurin series of the form

$$\psi_n(x)=\sum_{q=0}^{\infty}\gamma_q x^q, \quad \left(\gamma_q=\frac{\psi_n^{(q)}(0)}{q!}\right).$$

Since

$$S_n(\bar{a},\ k+1+\tau_1)=\frac{\Phi_n^{(k+1)}(0)}{(k+1)!},$$

equality (3) determines following recurrent formula for sequence $S_n(\bar{a}, r_i)$:

$$S_n(\bar{a},\ k+1+\tau_1)=\frac{1}{k+1}\sum_{j=0}^{k}S_n(\bar{a},\ j+\tau_1)\gamma_{k-j}. \qquad (4)$$

Thus, the calculation of a quantity of apexes/vertexes of the classes of equivalency is reduced to determination γ_q. Let us pause at the method of their determination.

Since

$$a_s \frac{x^{a_s-1}}{1+x^{a_s}} = \sum_{q=0}^{\infty} \varepsilon_{sq} x^q,$$

$$\varepsilon_{sq} = \begin{cases} 0, & \text{если}^{(1)}\ q \not\equiv a_s - 1 \pmod{a_s} \\ (-1)^{\left[\frac{q}{a_s}\right]} \cdot a_s, & \text{если}^{(1)}\ q \equiv a_s - 1 \pmod{a_s}, \end{cases}$$

Key: (1). if.

then

$$\gamma_q = \sum_{s=1}^{n} \varepsilon_{sq}. \qquad (5)$$

Thus, the algorithm of the consecutive determination of values $S_n(\bar{a}, r_\alpha)$ $(1 \leqslant \alpha \leqslant m)$ is unambiguously determined and is realized according to recursion formula (4), where coefficients γ_q are computed from formula (5). Let us note that $S_n(\bar{a}, r_\alpha) = 0$ when and only when $k \bar{\in} G$.

Page 123.

Assertion 2. The algorithm of the calculation of values $S_n(\bar{a} \cdot r_\alpha)$ makes it possible to describe all apexes/vertexes of set σ_α. In fact, all elements of set σ_α can be decomposed into tow types: elements/cells with component $x_n = 1$ (number of them is equal to $S_{n-1}(\bar{a}, r_\alpha - a_n)$ and elements/cells with component $x_n = 0$ (number of them is equal to $S_{n-1}(\bar{a}, r_\alpha)$). In this case

$$S_n(\bar{a}, r_\alpha) = S_{n-1}(\bar{a}, r_\alpha - a_n) + S_{n-1}(\bar{a}, r_\alpha). \qquad (6)$$

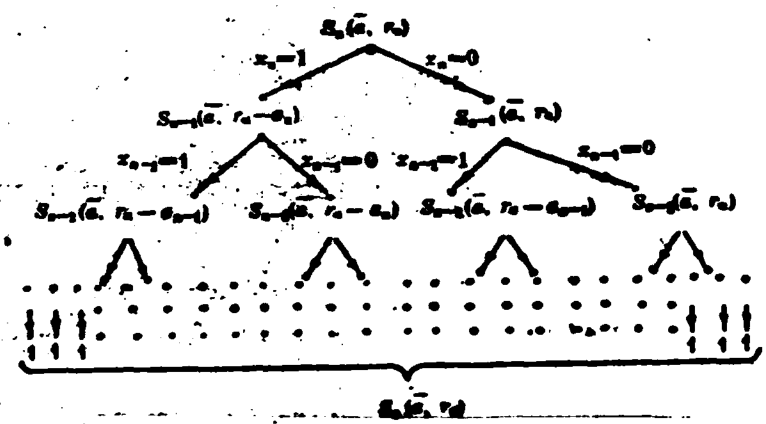
This separation of set σ_ϵ it is possible to represent thus:

$$\begin{array}{c} S_n(\bar{a}, r_\epsilon) \\ x_n=1 \swarrow \searrow x_n=0 \\ S_{n-1}(\bar{a}, r_\epsilon - a_n) \quad S_{n-1}(\bar{a}, r_\epsilon). \end{array}$$

We determine $S_{n-1}(\bar{a}, r_\epsilon - a_n)$ and $S_{n-1}(\bar{a}, r_\epsilon)$ through recursion formula (4): we find the values of components x_n of all elements of set σ_ϵ.

The analogous separation of sets $S_{n-1}(\bar{a}, r_\epsilon - a_n)$ and $S_{n-1}(\bar{a}, r_\epsilon)$ according to recursion formula (6) classes elements of set σ_ϵ to the subgroups on component x_{n-1}. Further, continuing thus the process of the separation of set σ_ϵ, we find all its apexes/vertexes.

As a whole the process of the separation of set σ_ϵ can be depicted in the form of the binary graph/count:



(1) Принцип нумерации элементов множества σ_ϵ

Key: (1). Principle of the numbering of elements of set.

Let $\bar{x}_i = (x_n, x_{n-1}, \ldots, x_1) \in \sigma_n$, $(i = 1, 2, \ldots, S_n(\bar{a}, r_n))$.

Page 124.

Value

$$p_i(\bar{x}_i, r_i) = x_n \cdot 2^{n-1} + x_{n-1} \cdot 2^{n-2} + \ldots + x_2 \cdot 2 + x_1$$

we will call the weight of this element/cell $\bar{x}_i \in \sigma_n$ $(i = 1, 2, \ldots, S_n(\bar{a}, r_n))$. All elements of set σ_n let us order by the decrease of their weights

$$p_1 > p_2 > \ldots > p_{S_n(\bar{a}, r_n)}$$

let us ascribe to each element/cell with a weight of p_v reference number $v (v = 1, 2, \ldots, S_n(\bar{a}, r_n))$.

Let us describe the algorithm of the determination of element/cell $\bar{x}_v \in \sigma_n$ from its preset number v_0.

We will use recursion formula (6):

$$S_n(\bar{a}, r_n) = S_{n-1}(\bar{a}, r_n - a_n) + S_{n-1}(\bar{a}, r_n).$$

Relatively components x_n of unknown element/cell $\bar{x}_{v_0} = (x_n, x_{n-1}, \ldots, x_1)$ with number v_0 are possible two cases:

$$x_n = 1, \text{ если}^{(1)} \; 0 < v_0 \leqslant S_{n-1}(\bar{a}, r_n - a_n);$$
$$x_n = 0, \text{ если}^{(1)} \; S_{n-1}(\bar{a}, r_n - a_n) < v_0 \leqslant S_{n-1}(\bar{a}, r_n).$$

Key: (1). if.

Relatively components x_{n-1} of unknown element/cell $\bar{x}_{v_0}$ with number v_0 are possible the cases

213

$$\text{при } x_n=1: \quad x_{n-1}=1, \text{ если } 0<\nu_0\leqslant S_{n-2}(\bar{a}, r_\alpha - a_n - a_{n-1}),$$
$$x_{n-1}=0, \text{ если } S_{n-2}(\bar{a}, r_\alpha - a_n - a_{n-1})<\nu_0\leqslant$$
$$\leqslant S_{n-1}(\bar{a}, r_\alpha - a_n);$$
$$\text{при } x_n=0; \quad x_{n-1}=1, \text{ если } S_{n-1}(\bar{a}, r_\alpha - a_n)<\nu_0\leqslant$$
$$\leqslant S_{n-2}(\bar{a}, r_\alpha - a_{n-1}),$$
$$x_{n-1}=0, \text{ если } S_{n-2}(\bar{a}, r_\alpha - a_{n-1})<\nu_0\leqslant$$
$$\leqslant S_{n-2}(\bar{a}, r_\alpha)$$

и т. д.

Key: (1). with. (2). if. (3). and sc forth.

Continuing this process, we find element/cell $\bar{x}_0 \in \alpha_n$, to which earlier was conferred number ν_0.

Let us describe the algorithm cf the formation of the number of this element/cell from set α_n.

It is obvious, number ν cf the preset element/cell consists within the limits

$$0<\nu<S_n(\bar{a}, r_\alpha).$$

Page 125.

The unknown number we will find by iterations $\nu_1, \nu_2, \ldots, \nu_n$ by the consecutive use of recursion formula (6).

First step/pitch of iteration. Depending on the value of component x_n of the element/cell in question are possible two cases. If $x_n=1$, then element/cell $\bar{x}_v$ belongs to set $S_{n-1}(\bar{a}, r_s-a_n)$. Then the unknown number consists within the limits

$$0<\nu<S_{n-1}(\bar{a}, r_s-a_n).$$

In this case first approximation ν_1 of number ν we assume/set by equal to zero.

If $x_n=0$, then element/cell $\bar{x}_v$ belongs to set $S_{n-1}(\bar{a}, r_s)$ and its number consists within the limits

$$S_{n-1}(\bar{a}, r_s-a_n)<\nu<S_n(\bar{a}, r_s).$$

In this case first approximation ν_1 of number ν will be determined by value

$$\nu_1=S_{n-1}(\bar{a}, r_s-a_n).$$

Second step/pitch of iteration. Depending on the first or second case of the previous iteration we realize respectively a separation of a number of elements/cells $S_{n-1}(\bar{a}, r_s-a_n)$ or $S_{n-1}(\bar{a}, r_s)$ and analogous with previous depending on the value of component x_{n-1} we determine the second approximation/approach of number ν_2, if $x_{n-1}=1$, then $\nu_2=\nu_1$; if $x_{n-1}=0$, then $\nu_2=\nu_1+S_{n-2}(\bar{a}, \tilde{k})$, where

$$\tilde{k}=\begin{cases} r_s-a_n-a_{n-1}, & \overset{(1)}{\text{если}}\ x_n=1 \\ r_s-a_{n-1}, & \overset{(1)}{\text{если}}\ x_n=0\ \overset{(2)}{\text{и т. д.}} \end{cases}$$

Key: (1). if. (2). and so forth.

Process is finished on i iteration ($1 \leqslant i \leqslant n$), if many elements/cells $S_{n-i}(\bar{a}, *)$ consist of one element/cell $\bar{x}$, in question then unknown number ν is equal to $\nu_i + 1$.

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Page 126.

USE~~[illegible]~~ OF COMPUTERS AND METHODS OF PREDICTION IN PROCESSES OF SUPPLY.

B. Kh. Nurkhaydarov, Ye. A. Pil'shchikov.

One of the most important branches of the national economy of Kazakhstan is the system of material and technical supply, which realizes systematic distribution of the production of production-engineering designation/purpose and its bringing/finishing from the supplier-enteprises to the user enterprises of all branches of material production.

The provisions of a majority of users with material resources in Kazakh SSR is made by a single system of the organs/controls of material and technical supply - by principal administration of the Council of Ministers of Kazakh SSR on the material and technical supply (by Glavsnab [- Main Supply Adminisration] of Kazakh SSR).

Entire activity of the organs/controls of supply of the system of Glavsnab of Kazakh SSR can be decomposed into the following stages.

1. Application operating period in course of which is revealed/detected necessity of national economy of Kazakhstan for production of production-engineering designation/purpose (according to nomenclature of GOSSNABS [- State Supply] of USSR).

2. Obtaining pools and distribution to their users.

3. Obtaining from Union-Republic ministries and departments of notices about earmarked pools into material resources with nomenclature of Gosplan of the USSR.

4. Specification and schedule-order of pools.

5. Realization and check of realization of assigned production both to enterprises and to bases UMTS; in necessary cases reception of measures for approximation/approach of delivery times.

6. Operational work, which includes assembly of information and representation in established/installed periods of data on realization of pools according to predicated nomenclature, method of

further claims from enterprises and their guarantee, extraction of further pools to enterprises and to organizations due to internal resources/lifetimes and decentralized purchase, operational solution of critical questions concerning guarantee of serviced users.

Page 137.

Thus, control process of the supply (deliveries) of production on broader scale consists of two phases:

a) the phase of planning/gliding (assembly of claims from the users, the composition of compound claim, obtaining pools, their distribution according to the users, the fastening of users to the suppliers), realized with the periodicity of times in half a year or block to the beginning of period being planned;

b) the phase of operational control, which ensures the realization of the initially comprised plan/layout of deliveries.

Operational control enters in the operation during the detection of deviations from the fulfillment of the plan of deliveries and consists in the generation of the solutions, directed toward the elimination of the disturbances/breakdowns of the normal running of deliveries to production.

By most critical and labor-consuming control function supply is the determination of necessity. The utilized at present methods of guaranteeing the necessity during the composition of the plan/layout of the material and technical supply of users possess a number of the deficiencies/lacks, caused by the following reasons: users compose claims long before the beginning of period being planned without the predicated plan/layout of production for the following year; necessities are overstated in view of the late introduction of changes of the norms of consumption into the card index of standard economy or consciously; users represent claims late.

Thus, the comprised in the appropriate levels of the system of Glavsnab compound claims of the users of region (republic) do not reflect real necessity. in this case the calculated necessity considerably differs from actual demand.

At the stage of the determination of the specified necessity taking into account the earmarked pools also appear large difficulties because many users send specification with the large delay or completely do not send. Thus, at the level of bases in similar situations it is necessary to realize a prediction/forecast of the specified necessity for the materials by the method of an increase in the necessity of last year by 10-15o/o, which unavoidably leads to the errors and, therefore, to the further work on the elimination of the emergent nonconformity.

The difficulties indicated lead to the need for using methods of the short term prediction, which have the relatively simple structure of model and at the same time which make it possible to obtain the reliable and sufficiently accurate results of the forecast of the necessity for material resources.

Page 128.

In this case it proves to be necessary to develop/process the methods, which ensure the consecutive updating of the forecasted characteristics taking into account the obtained deviations of the prediction/forecast of the necessity from the actual realization for the period, which precedes that planned/glided.

By very promising in this direction is use/application in the control the processes of the supply of the methods, worked out in the theory of the optimum control systems. Thus, for instance, analogous with systems with the accumulation of information about object [1] in control system by material and technical supply it is possible to observe the "prehistory" of the process of supply on the basis of the analysis of the available statistical account during the past periods, which will make it possible to more accurately consider the future values of the characteristics of this process, to replace the a priori probability distributions of characteristics with a posteriori ones and to make the best decisions in the beginning of each period being planned.

Furthermore, in control system by material and technical supply, which is characterized by the alternating phases of planning/gliding and operational control, it is desirable to even at the stage of planning/gliding take into account the dynamics of production on the supplier-enteprises and the dynamics of the consumption of resources/lifetimes by users in order to decrease in the certain degree the load on the system of operational control, since the account of these changes will make it possible to considerably reduce the probability of the appearance of disturbances/perturbations in the mode/conditions of operational control.

Let us introduce some designations. Let ρ_{ij} - quantity of service life of the i form, expended per unit time in accordance with the plan/layout of production and the norms of consumption, i=1, ..., N; j=1, ..., P. Then the integral curve of the consumption of the service life of the i form by certain set of user enterprises (for example, branch, region or of republic as a whole)

$$\varphi_i(t)=\sum_{j=1}^{p}\int_0^t p_{ij}(\tau)d\tau,\ i=1,2,\ldots,N, \qquad (1)$$

moreover at the end of the planned period (0, T) must be satisfied the condition

$$\varphi_i(T)=A_i,\ i=1,2,\ldots,N,$$

where A_i - output of the i form, assigned to the users of the specific branch, region or republic.

Page 129.

Thus, for the execution by the users of plan/layout up to moment/torque t suppliers must place not less $\varphi_i(t)$ the units of output of each form (i=1, 2, ..., N). In actuality sometimes occur delays and disruptions/separations of the plan/layout of deliveries due to the disturbances/perturbations in the system of material and technical supply (breakdown of aggregates/units on the supplier plant, changes of program or necessity of consumer plant, etc.). For the force of these reasons the actual curve of necessity $\tilde{\varphi}_i(t)$ differs from the planned conditions of use $\varphi_i(t)$ and it is certain random function of time.

Figure shows the sequence of the values of the realization of random function $\tilde{\varphi}_i$ at the particular moments of time, determined on the basis of the data of statistical account about the deliveries to production by the increasing result.

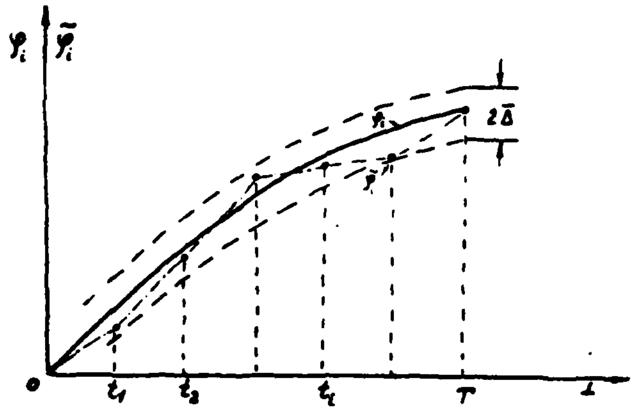
Let us consider certain economic process $\xi(t)$, under which, in particular, it is possible to understand the process of consumption at the level of individual enterprises, production associations, branches, economic regions or of republics.

Let it be process $\xi(t)=\xi_t$, changing discrete in the time, it is determined by the relationship/ratio

$$\Delta\xi_t=\xi_{t+1}-\xi_t,\ t=0,1,\ldots,T. \qquad (2)$$

Taking into account such economic characteristics, as the indices of the rates of growth $\tau_t=\frac{\xi_{t+1}}{\xi_t}$ and the increase $\lambda_t=\frac{\Delta\xi_t}{\xi_t}$, let us present relationship/ratio (2) in the form

$$\Delta\xi_t=\lambda_t\xi_t,\ t=0,1,\ldots,T. \qquad (3)$$



Page 130.

The solution of equation (3) for any moment t under the preset initial conditions (ξ_0, t_0) is determined by the formula

$$\xi_t=\left[\prod_{i=0}^{t-1}(1+\lambda_i)\right]\xi_0=\left(\prod_{i=0}^{t-1}\tau_i\right)\xi_0, \tag{4}$$

where $\tau_i=1+\lambda_i$.

In the case of the integral curve of the consumption of resource/lifetime it is logical to assume that $0\leqslant\lambda_i\leqslant 1$.

Virtually acceptable with this approach is the evaluation of the future values of demand (necessity) according to the predicted data of the rates of increase in the necessity, determined on the basis of the statistical analysis of information during the past periods.

Let us consider another approach to obtaining of the evaluation of the possible versions of the development of the necessity, based on the representation of the trajectory of process $\xi(t)$ in the form of the sum of regular component and the random process. Then for each particular moment of time $t_i (i=1, 2, \ldots, n)$ we have

$$\xi(t_i)=\xi^0(t_i)+\eta(t_i),\ i=1, \ldots, n, \tag{5}$$

where $\xi^0(t_i)$ - value of the trend of process $\xi(t)$ at moment/torque t_i;

$\eta(t_i)$ -- the value of the realization of the random component of process at moment/torque t_i.

Random component $\eta(t)$ can be considered as stationary random process with mathematical expectation $M[\eta(t)]=\text{const}$, by dispersion $\sigma_\eta^2(t)=\text{const}$ and correlation function $R_\eta(t_1, t_2)=R_\eta(\tau)$. Without the limitation of generality it is possible to assume $M[\eta(t)]=0$. We consider also that the component $\xi^0(t)$ can be presented in the form

$$\xi^0(t)=\sum_{j=1}^{\infty} a_j f_j(t), \tag{6}$$

where a_j - unknown parameters which must be determined,

$f_j(t)$ - the preset sequence of the linearly independent functions, for which is decomposed/expanded regular component.

In general form the task of determining the necessity for the following period can be formulated thus. Let be known the dynamics of the process of consumption $\xi(t)$ during the past periods: $\{\xi(t_i)\}, i=1, 2, \ldots, n$.

Page 131.

It is necessary to give the evaluation of demand $\xi_{n+1}=\xi(t_{n+1})$ at moment/torque $t_{n+1}=t_n+\tau_0$ (τ_0 - the time of prevention/advance). Forecasted value ξ_{n+1} depends on moments/torques t_i ($i=1, \ldots, n$) and from previously measured values $\xi_i, i=1, \ldots, n$, i.e.

$$\xi_{n+1}=f(\xi_1, \ldots, \xi_n; t_1, \ldots, t_n, t_{n+1}).$$

Under the assumption (5) by most advisable ones is considered the approach, based on the use of methods of the optimum filtration of random processes and sequences.

In our case the task consists in the isolation/evolution of regular component $\xi^0(t)$ from random $\eta(t)$, i.e., if is preset sequence in the form of the discrete/digital realization of the additive random process $\xi(t)$, which satisfies conditions (5) and (6), then it is required to filter out sequence $\xi(t_i), i=0, 1, \ldots, n$ at the moment of time $t_{n+1}=t_n+\tau_0$ then so that the dispersion σ^2_{n+1} of evaluation $m[\xi(t_{n+1})]$ would be minimum. By the methods of the theory of the filtration of random processes (sequences) [2] it is possible to determine the evaluations of the characteristics or process $\xi(t)$ at moment/torque t_{n+1}.

Let us register evaluation $\xi(t_{n+1})$ in the form

$$\hat{\xi}(t_{n+1})=\sum_{i=0}^{n} a_i \xi(t_i), \qquad (7)$$

where weight coefficients a_i are found from conditions the nondisplacement of the evaluation

$$M[\hat{\xi}(t_{n+1})]=M[\sum_{i=0}^{n} a_i \xi(t_i)]=\sum_{i=0}^{n} a_i \xi^0(t_i)=\xi^0(t_{n+1}) \qquad (8)$$

and of the minimum of the dispersion

$$\sigma^2_{n+1}=M[\xi^0(t_{n+1})-\hat{\xi}(t_{n+1})]^2=\sum_{i=0}^{n}\sum_{j=0}^{n} a_i a_j R(t_i-t_j). \qquad (9)$$

It is possible to show that the determination of optimum weights a_i is reduced to the task of minimization (9) during the limitations

$$\begin{cases} f_j(t_{n+1})=\sum_{i=0}^{n} a_i f_j(t_i) \\ (j=1,\ldots,m). \end{cases} \qquad (10)$$

Page 132.

With the help of the method of Lagrange's indefinite multipliers the extreme task indicated is reduced to the solution of the system of the linear equations relatively (n+1) of coefficients a_i and m of undetermined coefficients λ_k:

$$\begin{cases} \sum_{i=0}^{n} a_i R(t_i-t_j)-\sum_{k=1}^{m} \lambda_k f_k(t_j)=0 \\ \sum_{i=0}^{n} a_i f_k(t_i)=f_k(t_{n+1}),\ (j=0, 1,\ldots,n;\ k=1,\ldots, m). \end{cases} \qquad (11)$$

The dispersion of optimum evaluation we find through formula (9).

Thus, on the basis of the statistical analysis of information about the development of the necessity in the past and for the present is determined the trend of its development in the form of function (6) taking into account the obtained values of parameters a_i and is computed the value of demand when t_{n+1}.

The methods of the optimum filtration, which ensure the smallest variance of error, require the calculation of new weight coefficients for each point of realization. Furthermore, for guaranteeing a sufficient correctness of forecast at the preset length of discrete/digital realization can arise need in an increase in the frequency of the observation of process (for example, monthly or every ten days obtaining of summaries about the realization of deliveries), which leads to an increase in the volume of information. The solution of these problems is possible only in the presence of powerful/thick computational means (VTs of highest category at the level of central boards and single territoriat administrations). On the bases and territorial administrations, which have the limited computational power, it is expedient to use simpler (although are the less exact) methods of processing. To them can be attributed the methods of the slipping average, linear predictor, exponential smoothing (Braun's method) [3, 4], expert evaluations [5]. The required accuracy the evaluations of the parameters, and also the storage capacity and the high speed of computers determine the selection of one or the other method.

However, the methods of prediction cannot ensure the required accuracy and reliability in the determination of demand for the sufficiently prolonged periods due to the random factors, which appear in the system of supply and which lead to the uncertainty/indeterminancy in the prediction/forecast. Furthermore, information analysis according to a small number of observations does not make it possible to determine the effect of all these factors.

During the subsequent period can enter the operation also the concealed/latent previously factors, which to a considerable extent affect the development of demand and the reflecting specific shifts/shears in structure of consumption.

Page 133.

Thus, for instance, appearance of new users, change in the program of production in some enterprises and so forth lead to the need for making more precise and correcting the plan/layout, comprised in the previous stage of planning/gliding.

The greatest effect in the guarantee of connecting/fitting the program of production with the necessities for it all over the specified nomenclature can be achieved/reached in the case when is realized control of the process of supply (updating plan/layout) on the basis of making decisions in the beginning of period being planned (block, month) during the detection of the considerable deviations of actual necessity from the results of the forecast.

If in the stage of prediction is solved a question about that, what precisely versions of the trajectories of the future development of demand are essential and are subject to calculation (in the limits of confidence limits with the interval $2\bar{\Delta}$; see Fig.), then selection and realization of the best trajectory of the development of demand (in accordance with the selected criterion of optimum character and the limitations) can be realized with the help of the multistage process of making decisions, which makes it possible to carry out updating prediction/forecast taking into account to the current information about the deviations in the beginning of each period or within it through the fixed/recorded time intervals.

Assembly, processing and continuous accumulation of the required information, and also the solution on its basis of the complex of the tasks of control of supply (among other things of tasks of forecasting of necessity and periodic correction of the comprised plan/layout of supply) cannot be realized without the use of communications and computer the efficiency of functioning of which sharply grows upon their inclusion into the outline of ASU of the corresponding levels and components/links of the system of Glavsnab of KazSSR.

Let us pass to the construction of the model of the multistage process of making planning decisions in the control of supply. We consider that the system of supply has available the specific quantity of service lives in the form of reserve (supply) in order to have the capability at some moments of time $t_i \in (0, T)$, i=0, 1, ..., S-1 to realize a correction of the plan/layout of supply, allotting to users further pools in the decentralized order.

Let us designate through $\vec{u}=(u_1, \ldots, u_N)$ the vector of control; components u_i are the control pressures (value of the i form of resource/lifetime, isolated from the reserve), which ensure the correction of the vector of consumption $\vec{\xi}=(\xi_1, \ldots \xi_N)$, N - quantity of forms of the distributed production. A number of components of the vector of control can be less than N, if we take into account the interchangeability of some forms of production.

Page 134.

Let to the control be is superimposed the limitation in the form

$$\vec{u} \in \Omega(\vec{u}). \qquad (12)$$

Physically this means that the supplies cannot be unlimited. The optimum selection of boundaries of the region $\Omega(\vec{u})$ is given below.

Then trajectory of the process of supply (consumption curve) taking into account to the possibility of updating can be described by certain system of the differential equations

$$\frac{d\vec{\xi}}{dt}=\vec{f}(\vec{\xi}, \vec{u}, \vec{\eta}, t). \qquad (13)$$

Here $\vec{f}=(f_1, \ldots, f_N)$, moreover f_i - in general nonlinear functions from $\vec{\xi}, \vec{u}, \vec{\eta}$ and t; $\vec{\eta}=(\eta_1, \ldots, \eta_N)$ - vector of disturbances/perturbations whose components η_i are random processes.

For our purposes to conveniently examine process at the particular moments of time t_i $(i=0, 1, \ldots, S)$. Then the controlled object is described by the system of nonlinear difference equations in the vector form

$$\begin{cases} \vec{\xi}[k+1]=\vec{f}(\vec{\xi}[k], \vec{u}[k], \vec{\eta}[k], [k]), \\ k=0, 1, \ldots, S-1. \end{cases} \qquad (14)$$

As the reference input we will examine $\vec{\xi}^*(t)=(\xi_1^*, \ldots, \xi_N^*)$ - the vector function of the "full/total/complete" satisfaction of necessities, i.e., the function, which reflects the real necessity of joint user. Let us note that $(\vec{\xi}^*(t))$ - in general can be the random, unknown previously vector function of time. In this case $\vec{\xi}^*(t)$ we count the represented in the form sum of two components; regular function $\vec{\xi}_0^*(t)$ and stationary random process $\vec{\eta}$.

Page 135.

Then as one of the possible representations of the criterion of optimum character can serve the functional, which is the mathematical expectation of certain primary criterion Q_1:

$$Q = M\{Q_1(\vec{\xi}, \vec{\xi}^*, \vec{u}, \vec{\eta}, t)\} = \int_{\Omega(\vec{\eta})} Q_1(\vec{\xi}, \vec{\xi}^*, \vec{u}, \vec{\eta}, t) P(\vec{\eta}) d\Omega(\vec{\eta}), \quad (15)$$

where $\Omega(\vec{\eta})$ - range of change in the vector of disturbances/perturbations $\vec{\eta}$;

$P(\vec{\eta})$ - the probability density of random vector $\vec{\eta}$;

$$Q_1 = R[\vec{\xi}^*(t) - \vec{\xi}(t, \vec{u}(t))]$$

(R - certain convex function).

In the discrete/digital case we have

$$Q = M\{R(\vec{\xi}^*[k] - \vec{\xi}[k, \vec{u}[k]])\}. \quad (16)$$

Task consists of the determination of such strategy of control with which phase coordinates $(\xi_1, \xi_2, \ldots, \xi_N)$ and control pressures $(u_1, \ldots, u_N)$ would satisfy limitations, and preset criterion (16) reached the minimum. The posed above problem has two special features/peculiarities. The first lies in the fact that we should previously plan the size/dimension of the supplies, utilized subsequently in the process of updating the level of demand.

The second, chief characteristic of task consists in the insufficiency of initial (a priori) information relative to some parameters, entering the criterion of optimum character, or means of probability distribution for disturbance/perturbation $\vec{\eta}(t)$. If the type of the distribution of random process is preset, then can be unknown the parameters of this distribution (everyone ~ their part), and also equation of the controlled object, i.e., function f_i. Within the framework of the classical theory of optimum systems this task is irresolvable and the way of its solution lies/rests at the enlistment of adaptive approach [6, 1], which consists in the simultaneous study of object and the control of it. In this situation the control pressures carry dual character, since they serve not only for studying the object, but also for its reduction to the required state.

The solution of problem is divided/marked off into two basic stages.

Determination of limitations to control $\vec{u}$. let be known integral curve admissions of the productions of i form $\varphi_i(t)$ $(i=1, \ldots, N)$ from the supplier, which can be determined on the basis of prediction/forecast or information about the schedule chart of the deliveries, worked out by supplier. Let $\psi_i(t)$ - integral curves of necessities, calculated, for example, with the help of the methods of prediction.

As a result of the incomplete agreement of graphs $\phi_i(t)$ and $\varphi_i(t)$ in the time products delivery to users will be delayed to certain value ε_i.

Page 135.

This delay in the fulfillment of plan is equal to

$$\varepsilon = \max_{1<i<N}(\varepsilon_i, 0), \qquad (17)$$

where ε_i - minimum delay, caused by the absence of the necessary output of the i form. If functions φ_i and ψ_i are preset in analytical form, then value ε_i is found [7] from the equation

$$\varphi_i[t(\varepsilon_i)-\varepsilon_i]-\psi_i[t(\varepsilon_i)]=0, \ (i=1, 2, \ldots, N). \qquad (18)$$

Here $t(\varepsilon_i)$ indicates the moment/torque of time t, with which function $\varphi_i(t-\varepsilon_i)-\psi_i(t)$ reaches maximum.

During the graphic assignment of curves $\varphi_i(t)$ and $\psi_i(t)$ minimum is easily determined by the simple shift/shear of graph $\varphi_i(t)$ along axis t.

We consider that the order of output does not vary. In this case so that the users would avoid considerable losses $F(\varepsilon)$ in the case of the delay of delivery time, it is expedient to have supplies according to each i form of production (i=1, 2, ..., N).

Knowing $\psi_i(t)$, taking into account supplies, and also $\varphi_i(t)$ we determine with the help of (18) delay $\varepsilon_i(u_i)$ in the form of monotonically decreasing function from u_i.

Thus, task consists in determination $u_i^*\geqslant 0$, of those minimizing the total losses

$$R=F[\max_{1<i<N}\varepsilon_i(u_i)]+\sum_{i=1}^{N}c_iu_i, \qquad (19)$$

(c_i - expenditure for creation and storage of the unit of output of the i form).

The algorithm of the solution of this problem makes it possible to obtain the evaluations of limits $\{u_i^*\}$ $(i=1, 2, \ldots, N)$ of the permissible range of change in the control pressures, in this case the error in the determination of these values depends on the accuracy of the results of the forecast of curve φ_i. P Multistage process of making adaptive type decisions. For the digital processes of form (14) in general the solution of the task, which consists in the minimization of the function of criterion (16) taking into account the restrictions placed on control pressures $u_i=(u_i^0,\ldots,u_i^{n-1})$

$$\sum_{k=0}^{n-1}u_i^k<u_i^*,\ (i=1,\ldots,N) \qquad (20)$$

presents considerable difficulties.

Page 137.

Therefore it is expedient to do a series/row of the simplifying assumptions relative to model (14), (16), (20).

We will consider that the structure of functions f_i is known and phase coordinates ξ_i do not depend on the variable/alternating for which j≠i (i, j=1, 2, ..., N). Then relationship/ratio (14) can be presented in the form

$$\begin{cases} \xi_i[k+1]=f_i(\xi_i[k], u_i[k], \eta_i[k]), \\ (i=1, \ldots, N; \ k=0, 1, \ldots, S-1). \end{cases} \tag{21}$$

Lowering index i, it is possible to register dependence (21) symbolically in the form of operator F, i.e., in the form of certain stochastic conversion, which describes the transition of system from one discrete/digital state to another, namely

$$\xi_{k+1}=F(\xi_k, u_k, \eta_k), \ k=0, 1, \ldots, S-1. \tag{22}$$

Here ξ_0=c - initial state of system, η_k - independent random quantities with the known type of the function of distribution $dG(\eta)$, but with the unknown parameters. For example, if η_k are subordinated to the normal law of distribution, then by the unknown parameters there can be the mathematical expectation m and dispersion σ^2.

Furthermore, at each moment of time k (k=0, 1, ..., S-1) to us is known only evaluation $\hat{\xi}_k^0$ of regular component of process ξ_k, of that obtained on the basis of prediction/forecast.

If we consider $\xi(t)$ process additive and represented in the form (5), then at each step/pitch of making decision we have two unknown parameters ξ_k^0 and $\sigma^2(\eta_k)$, since constant $m(\eta_k)\neq 0$ can be included/connected in regular component $\xi^0(t)$.

The criterion of optimum character (16), connected with the multistage process of making decisions, let us present in the form

$$Q=M\{R_s\}=M\{\sum_{k=0}^{s-1}r_k(\xi_k,u_k,\eta_k)\}. \qquad (23)$$

At the first step/pitch as the a priori information we have a distribution $dG(\eta)$. Having available the systematic procedure of the modification of this a priori function of the distribution (for example, realizing a prediction/forecast of the parameters on the basis of analysis and information processing about the past and present), we can at the following step/pitch obtain the new function of distribution $dG_1(\eta)=T(\eta, c, u_1, \eta_1, G)$, which depends on the previous function of distribution G, from value η_1, of that obtained from the observations, from the initial state $\xi_0=c$, new state $\xi_1=F(c, u_1, \eta_1)$ and solution u_1.

Page 138.

Let $f_s(c, G)$ - mathematical expectation R_s, obtained with the optimal strategy, initiated in the state (c, G), then on the basis of the principle of Bellman's optimum character obtain the following functional equation:

$$f_S(c, G) = \min_{u_1} \left\{ \int [r(c, u_1, \eta_1) + f_{S-1}(F(c, u_1, \eta_1), G_1)] \times \right.$$
$$\left. \times \, dG(\eta_1) \right\} \qquad (24)$$

for all $S \geq 2$; for S=1

$$f_1(c, G) = \min_{u_1} \left[\int r(c, u_1, \eta_1) dG(\eta_1) \right]. \qquad (25)$$

As the procedure of the modification of a priori information it is possible to use other methods, for example known in the probability theory bayes' formula.

Thus, work shows the possibility of using the adaptive approach with the solution of the problems of planning/gliding and management of the process of the supply, which makes it possible to be rejected (in full or in part) from the existing methods of planning/gliding (determination of compound necessity via the assembly of claims from the users) and of employee by instrument for the consecutive evaluation and the correction of the parameters of the development of the necessity within the interval [0, T] for the process of making decisions under conditions of incomplete information.

The model examined is solved with the help of the methods of the functional equations of dynamic programming. Important moment/torque is use at the higher levels of control of the supply of the means of computer technology, which make it possible to accumulate and to operationally treat the considerable volumes of statistical information, which becomes in principle irresolvable task during the time constraints of the solution without the use by computers and ASU.

This material makes it possible to plan the directions of further works in this region.

Page 139.

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Page 140.

One numeration system with the recurrently determined weights [1].

FOOTNOTE [1]. Work is carried out under I. Ya. Akushskiy's management/manual. ENDFOOTNOTE.

I. T. Pak, A. D. Kharlip.

Despite the fact that the positional methods of representation of numbers with the natural weights of digits and close to them systems have a series/row of obvious advantages, at present are continued the searches for the new methods of representation of the numbers which would make it possible to achieve one or the other advantages during the construction of machine arithmetic.

When selecting of the bases (weights) of numeration system arises interesting question. Is it possible so to select bases in order to decompose numbers into the groups of digits and during this separation during the addition it would not be the transfer from the group into the group? Apparently, the possibility of this separation is concealed in the ambiguity of the representation of a number in this system, which is determining by the concrete/specific/actual properties of bases.

Let us consider the nondecreasing sequences of natural numbers $\{x_i\}$ with $x_1=1$.

According to Hoggatu and King [1], the sequence of positive integers $\{x_i\}$ is called full/total/complete, if each natural number n has a representation in the form

$$n=\sum_1^N c_i x_i,$$

where $c_i \in \{0, 1\}, N$ - sufficiently large.

Hence it is apparent that if we should present any natural number n in the form of the sum of different members with coefficients c_i of certain preassigned sequence $\{x_i\}$, thenthis is possible only if this sequence is full/total/complete.

Is known theorem [2], which gives necessary and sufficient because of completeness of arbitrary sequences.

Theorem. Let $\{x_i\}$ be the nondecreasing sequence of positive integers with $x_1=1$.

Page 141.

Sequence $\{x_i\}$ will be full/total/complete when and only when is satisfied the condition

$$x_{p+1} \leqslant 1+\sum_{1}^{p} x_i,\ p=1,2,\ldots \qquad (1)$$

During the construction of numeration system with the artificial weights the sequence of weights $\{x_i\}$ necessarily must satisfy condition (1).

It should be noted that condition (1), guaranteeing the possibility of using the sequence for the representation, yet does not guarantee the uniqueness of this representation. The conventional numeration systems use as the sequence of weights geometric progressions with different denominators. It is not difficult to ascertain that they all satisfy condition (1), which for them takes the form

$$x_{p+1}=1+\sum_{1}^{p} x_i,\ p=1,2,\ldots \qquad (2)$$

The transition of condition (1) under condition (2) is the basis of the fact of the uniqueness of the representation of numbers in the ordinary binary number system.

In general with execution (1) the uniqueness of representation is broken; however, questions of the evaluation of a number of representations for the arbitrary full/total/complete sequences are worked out still insufficiently. There is only a series/row of results for the concrete/specific/actual sequences, for example for the sequences of Fibonacci and Lyuk [3, 4].

Does arise question, it is not possible to use the full/total/complete sequences, which possess the property of the nonuniqueness of representation, for simplification in some operations.

In this article is done the attempt to use nonuniqueness of representation for the construction of the positional numeration system, which possesses the properties, close to the properties of the nonpositional numeration system. In particular, is done the attempt to attain the independence of operation with the addition with the groups of digits.

In this direction it seems to us natural to use as the sequence of artificial weights such full/total/complete sequences which remain full/total/complete after the dropping of them of certain number of terms. It is obvious that such sequences exist, but not of any full/total/complete sequence it is possible to recede, without breaking its completeness, any number of terms. Thus, for instance Fibonacci's, sequence ceases to be full/total/complete after the omission from it of two arbitrary members [5].

Page 142.

Let us construct the concrete/specific/actual full/total/complete sequence, which possesses the properties indicated above:

$$\{\ldots, 460, 273, \boxed{187}, 115, 72, 43, \boxed{29}, 18, 11, 7, \boxed{4}, 3, 1, 2\}. \quad (3)$$

It is determined from the following recurrent relationships/ratios:

$$x_1=2,\ x_2=1,$$
$$x_{4n-1}=x_{4n-2}+x_{4n-3},$$
$$x_{4n}=x_{4n-1}+x_{4n-2},$$
$$x_{4n+1}=\sum_1^{4n-1} x_k+1,\ k\neq 4i,\ i=1, 2, \ldots,$$
$$x_{4n-2}=x_{4n-3}+x_{4(n-1)}.$$

The special feature/peculiarity of the construction of sequence (3) consists in the fact that the dropping of it of each fourth term does not break its completeness.

$$\{\ldots, 733, 460, 273 \square 115, 72, 43 \square 18, 11, 7 \square 3, 1, 2\}. \quad (4)$$

Let us name sequence (4) in contrast to sequence (3) sequence with the "windows", and each set of three of terms, arranged/located between (k-1) -th and the k window - by k triad.

Let us demonstrate the completeness of sequence (4). For this it suffices to establish that for any term of it is satisfied the condition (1). Sequence (4) can be constructed without depending on sequence (3) according to the formulas

$$z_1=2,\ z_2=1,$$
$$z_{3n}=\sum_{3n-2}^{3n-1} z_k,$$
$$z_{3n+1}=\sum_{1}^{3n} z_k+1, \qquad (5)$$
$$z_{3n+2}=\sum_{3n-1}^{3n+1} z_k.$$

From formulas (5) it is possible to obtain the following relationships/ratios:

$$z_{3n}<\sum_{3n-2}^{3n-1} z_k+1,$$
$$z_{3n+1}=\sum_{1}^{3n} z_k+1,$$
$$z_{3n+2}<\sum_{3n-1}^{3n+1} z_k+1.$$

Page 143.

Thus, condition (1) is satisfied for all elements/cells of sequence (4).

Sequences (3) and (4) subsequently are used by us as the bases of binary number system.

Under the range of the representation of numbers in this system, as usual, we will understand a quantity of different numbers which can be represented in this system. Thus, if we take as the artificial weights for the representation of numbers n of the members of sequence (3), then range will be equal to the sum of these n of terms. This sum will be that quantity of different numbers which we can present in the preset range. It is logical that during the use as the weights of sequence (4), the range of the representation of numbers is reduced by the sum of members with the numbers $4n$.

The representation of numbers in numeration system with weights (3) we will call surplus representation, and in numeration system with weights (4) - normalized.

Let us consider the operation of addition. The addition of any two numbers, which have the normalized representation, can be led with the help of the given tables of triad addition.

Table 1 is used for adding the numbers, represented in the range of the first triad. Table 2 gives the results of adding those numbers which are already represented in the range of the first and second triads (the first triad consisting of zero), and also with the help of any n of the triads, the first n-1 of which consist of zero.

Table 1.

+	010	001	100	110	101	111
010	001	100	110	101	111	1100
001	100	110	101	111	1100	1110
100	110	101	111	1100	1110	1101
110	101	111	1100	1110	1101	1111
101	111	1100	1110	1101	1111	11000
111	1100	1110	1101	1111	11000	11010

Page 144.

From the tables it is evident that the results of addition are obtained in excess representation. Therefore the operation of addition must be realized into two stages. Through the table we at first find sum in surplus representation, then we translate it into the normalized representation by normalization.

Example. To sum numbers 150 and 64.

115	72	43	$\boxed{29}$	18	11	7	$\boxed{4}$	3	1	2	— (1) базисы системы
1	0	0		1	1	0		1	1	1	— 150
0	0	1		1	0	0		1	0	0	— 64
1	0	1	$\boxed{1}$	1	0	0	$\boxed{1}$	1	0	1	— (2) результат в избыточном представлении
1	1	0		1	0	1		0	0	1	— (3) результат в нормализованном представлении

Key: (1). the bases of system. (2). result in surplus representation. (3). result in normalized representation.

A process of normalization can be carried out with the help of the series/row of the further formulas:

$$x_{4n+1}=x_{4n-1}+x_{4n-2}+2x_{4n-3},$$

$$x_{4n+1}=x_{4n}+x_{4n-1}+x_{4(n-1)},$$

$$2x_{4n+1}=x_{4n+2}+\sum_{j=1}^{n-1}x_{4n+2-4j}+x_{2},$$

$$2x_{4n-1}\doteq x_{4n-1}+x_{4n-2}+x_{4n-3},$$

$$2x_{4n-2}=x_{4n-1}+x_{4n-5}+x_{4n-6}.$$

Table 2.

+	001	010	100	101	110	111
001	$100\bar{1}$	100	101	$111\bar{1}$	111	$1100\bar{1}$
010	100	$11\bar{1}$	110	111	1010	1100
100	101	110	111	$1100\bar{1}$	1100	$1110\bar{1}$
101	$111\bar{1}$	111	$1100\bar{1}$	$1101\bar{1}$	1101	$1111\bar{1}$
110	111	1010	1100	1101	1110	1111
111	$1100\bar{1}$	1100	$1110\bar{1}$	$1111\bar{1}$	1111	11000

Page 145.

It is possible to propose another method of normalization which easily is realized on the computers.

We find all representations of zero in the ternary system in the preset s-band the use as the bases of sequence (3), and as the bases/bases { 1, 0, -1 }. Among them always will be located such representations, which during the addition with surplus representation of a number give its normalized representaton.

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Page 146.

One model of compression, realized on the computers of the type M-20.

I. T. Pak, V. V. Satsuk.

In the previously published work [1] was examined a question about the compression of the numerical information, organized by its storage in the form of label on the positional ruler.

FOOTNOTE [1]. I. Ya. Akushskiy, I. T. Pak. To a question of the organization of storage devices/equipment TsVM on the principle of the compression of information. "Bulletin of AS of KazSSR, series physico-mathematical", 1971, No 5. ENDFOOTNOTE.

Using this idea based on the example of the multiplication of matrices/dies 54x54, constructed model the compressions of information for the realization on computers of the type M-20. With the ordinary method only for storing the initial and resulting information it is necessary to expend $3x54^2$=8748 45 bit information words, which exceeds the possibility of the working storage of computers of the type M-20.

As the basis of model are assumed the procedures, named by us the "scale" and the "search" whose block diagrams are given in figures 1 and 2. Procedures "scale" and "search" ensure the dispatching of numbers to the positional ruler, sample of it and readdressing in the address part of the program after each change the numbers of label on the positional ruler.

Description of algorithm. Let us isolate for organizing the "position storage" of h 45-bit information words. Since we will use with positive and negative numbers, then numbers of initial and resulting information must satisfy the condition

$$|x_m|<\frac{1}{2}(45\times h),$$

where $m=1,2,3,\dots,8748$ - total quantity of numbers, which are brought in to the positional ruler.

Page 147.

Entering the positional ruler number x_m is designated k and p; k indicates the number of instruction, and p - address, in which will be preserved the number of number x_m. The variable/alternating i (Fig. 2), passing value from -1/2 (45Xh) to +1/2 (45Xh), it determines the number of number x_m depending on the values of values, previously come position ruler, after this occurs the full/total/complete or partial renumbering of the previous values.

The restoration/reduction of number x_m occurs as follows. Being converted to the procedure "search" (Fig. 1), we are given index f, which indicates the number of instruction, and the index γ, which indicates the number of address.

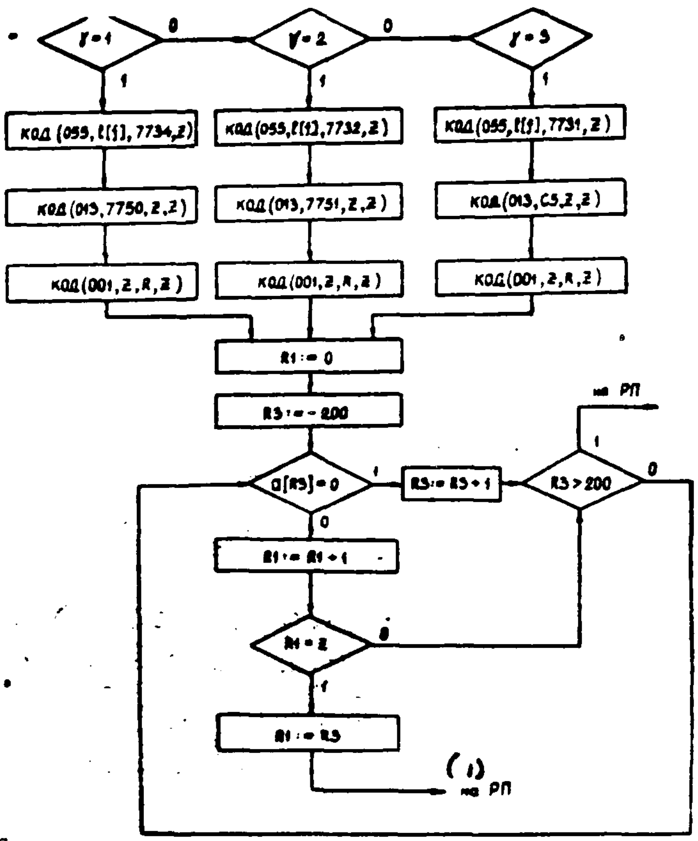

Fig. 1. Block diagram of procedure "search (f, γ, R1)".

Key: (1). on.

Page 148.

Then we form/shape number z, which is determining the number of number z_m, and on the positional ruler we from left to right seek z-y bits, not equal to zero. Value x_m is defined as the sum of zero and single digits.

The multiplication of matrices/dies occurs as follows. Each line of initial matrix/die elementwise with the help of the procedure "scale" is placed in the position ruler. After this with the help of the procedure "search" is conducted the sample of equivalent components of matrices/dies for the formation of the elements/cells of the resulting matrix/die.

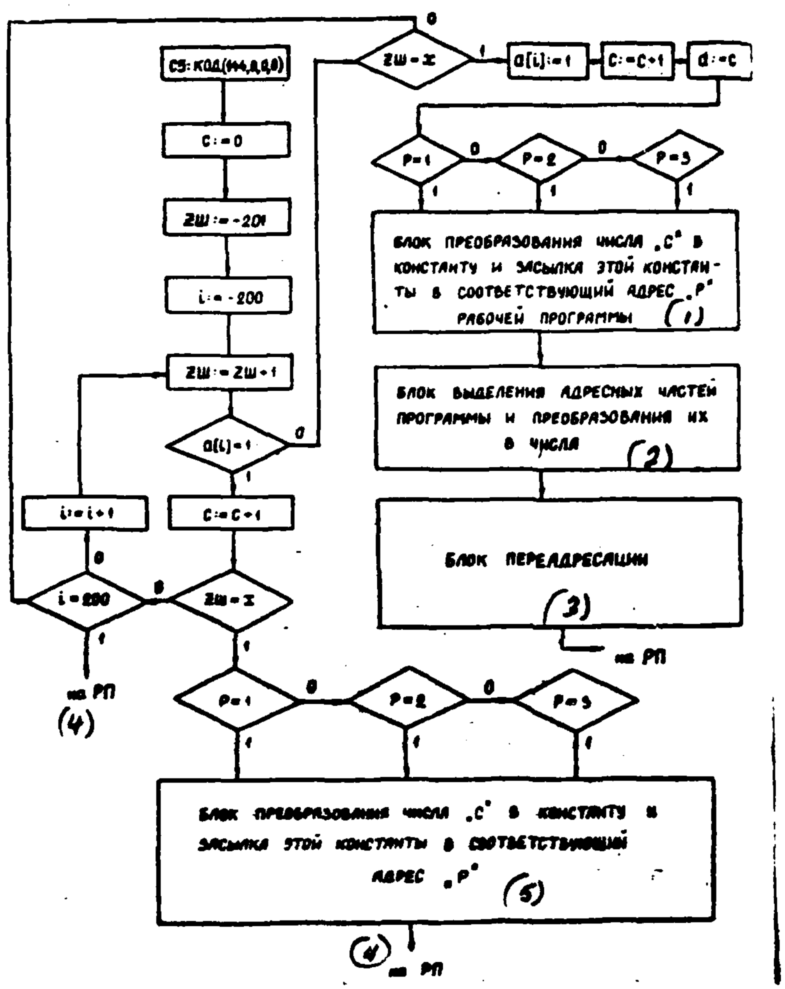

Fig. 2. Block diagram of procedure "scale (x, k, p)".

Key: (1). Unit of converting the letter "c" into the constant and the dispatching of this constant in the appropriate address "R" of working program. (2). unit of extraction of address parts of program and their conversion into numbers. (3). unit of readdressing. (4).

on. (5). unit of converting number s "" into constant and dispatching of this constant in appropriate address "r".

Page 149.

The formed line of the resulting matrix/die with the help of the procedure "scale" is placed in the position ruler.

All elements/cells of three matrices/dies can be given out to the printing with the help of the procedure "search" and are used for further operations.

In the model as the digit of position storage is used full/total/complete information word. In spite of this, for the realization of algorithm were required only 3100 information words.

Page 150.

SIMULATION OF THE RELIABILITY OF THE ELEMENTS OF EXCHANGE SYSTEMS BY INFORMATION ON TSVM.

I. G. Pil'shchikova.

During the simulation of the processes of the functioning of exchange systems by data considerable effect on the results proves to be the account of the interaction of environment and, in particular, reliability of the functioning of the elements of system. We have examined the series/row of the imitation models of elements/cells, which are restored in the interval of simulation, and the communication networks as a whole, which can serve as background for the reproduction of transmittings of information taking into account different methods of increasing the correctness.

Model 1 reflects the steady functioning of the one-way channel of communications without the redundancy. It considers the flows of failures and restorations/reductions of channel, and also the intervals of its entrance into the synchronism after failures. In the given in figure 1 graph/count of transitions the apexes/vertexes correspond to the states of the communication channel: apex/vertex 1

- to state of readiness for the transmission of information, apex/vertex 2 - to state of the failure of the communication channel and apex/vertex 3 - to state of pulling into step of the restored/reduced communication channel, and edges/fins - to allowed transitions λ, μ and θ are the indices of failures, restorations/reductions and connection respectively and characterize the reasons for transition of one state to another.

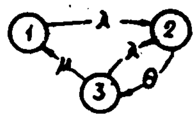

Fig. 1.

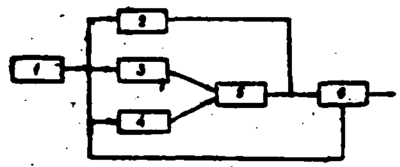

Fig. 2.

Page 151.

The algorithm of the work of model is represented on figure 2. It is based on the imitation of three parallel processes: the production/consumption/generation of the moments/torques of failures 2, restorations/reductions 3 and termination of pulling into step 4. The enumerated units are organized as random-number generators, distributed according to the appropriate laws. Tuner 1 defines concretely the parameters of the communication channel which must be imitated.

Since the states of restoration/reduction and pulling into step are nonoperative for the channel, occurs fixation of the time intervals of these states and is counted the coefficient of the lack of preparation (employment) of the channel of communication (unit 5).

The work of model is limited to the assigned time of simulation $t_{кон}$, analysis of which is produced by unit 6.

The interpretation of model in the language of SLANG-system is given below.

Канал без резерва (1)

```
BEGIN
INTEGER TKOH, T, T1;
REAL L1, L2, L3; FACILITY КАНАЛ; (2)
INPUT (TKOH, L1, L2, L3);
PROCESS OTKA3; BEGIN T: =EXPONENT (1/L1);
OUTPUT (L1, L2, L3, TKOH);
NEW OTKA3 TO N1; WAIT TKOH; STOP;
N1: =WAIT T; NEW BOCCT TO M 1; END;
PROCESS BOCCT; BEGIN CANCEL; M1: SEIZE КАНАЛ; (2)
WAIT EXPONENT (1/L2); RELEASE КАНАЛ; (2)
NEW ПОДКЛ TO P1; END;
PROCESS ПОДКЛ; (3) BEGIN CANCEL; P1: T: =EXPONENT (1/L1);
T1: =EXPONENT (1/L3); SEIZE КАНАЛ; (2)
WAIT IF T>T1 THEN T1 ELSE T;
RELEASE КАНАЛ; (2)
IF T>T1 THEN NEW OTKA3 TO N1 (4)
ELSE NEW BOCCT TO M1; END END
```

Key: (1). Channel without the reserve. (2). channel. (3). switch-on. (4). failure.

Model 2 imitates the steady functioning of the routing, which contains to three channels with floating redundancy: by 2/1, 1/1 and 1 channel without the reserve. It is the more general-purpose modification of the model, examined earlier [3].

Model reproduces the independent flows of failures and restorations/reductions of the communication channels, and also the intervals of connection and pulling into step with the development/detection of nonoperative intervals and probability of the lack of preparation of each of the communication channels.

The graph/count of transitions with one channel without the reserve is analogous given above in the examination of model 1. In the case of the simulation of direction with the version of redundancy 1/1 graphs/counts of transitions take the form, represented in figure 3.

Page 152.

In this graph/count the apex/vertex 00 corresponds to the state of the failure of two communication channels, 40 - to state when the first channel enters into synchronism, the second - in the failure, 10 - the first channel conducts the transmission of information, the second in the state of failure, 12 - the first channel conducts transmission, and the second is in the state of readiness, 42 - the first channel enters into the synchronism, the second in the state of readiness, 04 - first channel in the failure, the second enter into synchronism, 01 - first channel in the state of failure, and on the second is conducted the transmission of information, 21 - first channel in the state of readiness, the second conducts transmission, 24 - first channel in the state of the readiness, the second enter into synchronism.

The behavior of routing with redundancy 2/1 is described by the graph/count of states with 31 apex/vertex even ,20 by edges/fins. To avoid the interpretation of the number of each of the apexes/vertexes and reproductions of transitions in the graph/count to each state (apex/vertex) are set in the conformity three octal digits, each of them (from left to right) positional designates the following: the first - what channels are located on the stage of pulling into step; the second - what communication channels they are found in the readiness: the third - what communication channels conduct the transmission of information.

During their recording in the binary code in each of high-orders digit relates to the channel of communication No 1, the following - to the channel of communication No 2, low-order - to the channel of communication No 3. For example, state 431 means that (first digit - 100) the channel of communication No 1 enters into synchronism, furthermore (second digit -011), the channels of communication No 2 and No 3 are found in the readiness, but (third digit - 001) transmission is conducted according to channel No 3.

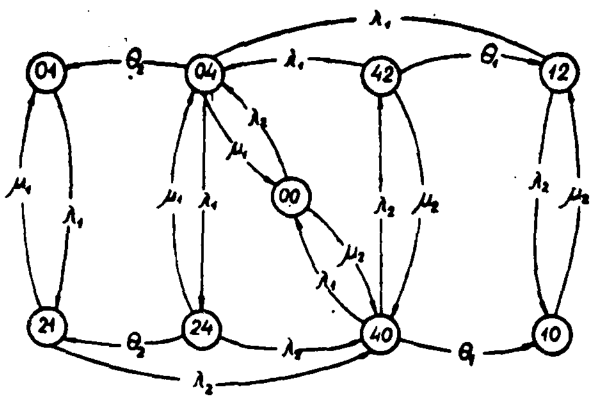
01
04
42
12
00
21
24
40
10
θ_2
λ_1
θ_1
λ_2
μ_1
μ_2

Fig. 3.

Page 153.

Причины (2) / Исходное состояние (1)	λ_1	λ_2	λ_3	μ_1	μ_2	μ_3	θ_1	θ_2	θ_3
000	400	200	100						
011	411	211				000			
022	422		122		000				
033	073				011	022			
044		242	144	000					
055		075		011		044			
066			076	022	044	0			
073				033	411	422			
075				211	055	244			
076				122	144	066			
100	500	300				000			011
122	162				100	022			033
144		164		100		044			055
162				122	500	422			073
164				300	144	244			075
200	600		300		000			022	
211	251				011	200		033	
244			254	200	044			066	
251				211	411	600		073	
254				300	144	244		076	
300	340				100	200		122	211
[illegible]				300	500	600		152	251
400		600	500	000			044		
411		[illegible]		011		400	055		
[illegible]			[illegible]	[illegible]	400		066		
[illegible]				211	411	600	075		
[illegible]				122	500	422	076		
[illegible]		[illegible]		100		400	144		411
[illegible]				300	500	600	164		431
600			610	200	400		244	422	
610,				300	500	600	254	432	

Key: (1). Initial state. (2.) Reasons.

Page 154.

The graphic representation of all possible transitions in this case is hindered/hampered; therefore they are represented by table.

The block diagram of the operation algorithm of model is represented in figure 4.

After the setup of model for preset parameters 1 and the assignment of the version of redundancy 2 is organized the parallel imitation of the functioning of each of three channels of communication (4.6.8). The work of each of the models of the communication channel is analogous to the work of model 1.

For the version of reserve 1/1 and 1 to unusable communication channels is written/recorded the "employment" to the moment/torque of the termination of simulation. However, in the case of redundancy 2/1 work all three channels.

The analysis of the need for the connection of the spare communication channels makes it possible to carry out a replacement of the channel, out-of-order, by finished spare channel with the delay to the period of connection (unit 9). This time is nonoperative interval and together with the intervals of restoration/reduction and pulling into step is fixed/recorded in appropriate counters (3.5.7).

The work of model is finished at the moment of time $t^{мод}_{кон}$, provided by assignment (unit 10).

The interpretation of program in the language of SLANG-system is represented below.

Направление связи (1)
(не более трех каналов)

```
BEGIN
INTEGER A1, PE3, A, B, C, K1, K2, K3, K, T1 [K], T2 [K], T3 [K];
REAL L1 [K], L2 [K], L3 [K];
FACILITY КАНАЛ [K]; STATISTIC КАНАЛ; (2)
INPUT (A1, PE3, K, L1, L2, L3);
PROCESS OTKA31;
```

Key: (1). Routing (not more than three channels). (2). channel.

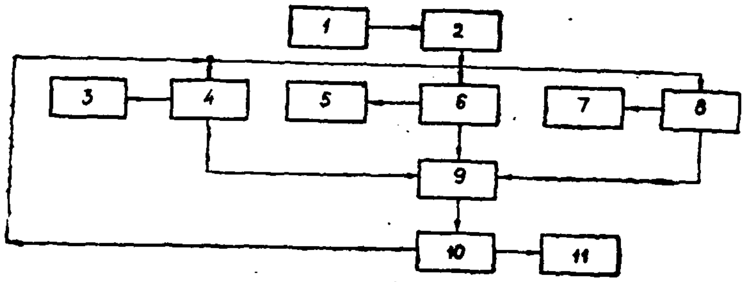

Fig. 4.

Page 155.

```
BEGIN
NEW OTKA31 TO F1; WAIT A1; STOP;
F1:
K1:=1; K2:=2; K3:=3;
GOTO:(S1, S2, S3), PE3;
S1: T1[K2]:=A1; T1[K3]:=A1; NEW OTKA31 TO S11;
SEIZE KAHAЛ [K2]; WAIT A1;
S11: C:=1; B:=0; NEW OTKA31 TO N11;
SEIZE KAHAЛ [K3]; WAIT A1;
S2: T1 [K3]:=A1; C:=1; B:=0;
NEW OTKA31 TO M12; NEW OTKA31 TO N11;
SEIZE KAHAЛ [K3]; WAIT A1;
M12: SEIZE KAHAЛ [K2]; GOTO M11;
S3:
NEW OTKA31 TO M5;
NEW OTKA31 TO P5;
N11: A:=1;
T1 [K1]:=EXPONENT (L1 [K1]);
N: WAIT T1 [K1]; A:=0; SEIZE KAHAЛ [K1];
N1: T2 [K1]:=EXPONENT (L2 [K1]); WAIT T2 [K1];
IF B CON C THEN WAIT UNTIL (NO B) DIS (NO C);
T1[K1]:=EXPONENT (L1[K1]);
N2: T3[K1]:=EXPONENT (L3[K1]);
WAIT IF T1[K1]>T3[K1] THEN T3[K1] ELSE T1[K1];
IF T1[K1]>T3[K1] THEN GOTO N3 ELSE GOTO N1;
N3: A:=1; RELEASE KAHAЛ [K1]; GOTO N;
N4: WAIT UNTIL (NO B) DIS (NO C); T1[K1]:=EXPONENT (L1[K1]);
WAIT IF T1[K1]>T1[K2] THEN T1[K2] ELSE
IF T1[K1]>T1[K3] THEN T1[K3] ELSE T1[K1];
IF T1[K1]>T1[K2] THEN GOTO N2 ELSE IF T1[K1]> T1[K3]
THEN GOTO N2 ELSE GOTO N1;
M5: B:=1;
T1[K2]:=EXPONENT (L1[K2]);
M: WAIT T1[K2]; B:=0; SEIZE KAHAЛ [K2];
M1: T2[K2]:=EXPONENT (L2[K2]); WAIT T2[K2];
M11:
IF A CON C THEN WAIT UNTIL (NO A) DIS (NO C);
T1[K2]:=EXPONENT (L1[K2]);
M2: T3[K2]:=EXPONENT (L3[K2]);
WAIT IF T1[K2]>T3[K2] THEN T3[K2] ELSE T1[K2];
IF T1[K2]>T3[K2] THEN GOTO M3 ELSE GOTO M1;
M3: B:=1; RELEASE KAHAЛ [K2]; GOTO M;
M4: WAIT UNTIL (NO A) DIS (NO C); T1[K2]:=EXPONENT (L1[K2]);
WAIT IF T1[K2]>T1[K1] THEN T1[K1] ELSE
IF T1[K2]>T1[K3] THEN T1[K3] ELSE T1[K2];
IF T1[K2]>T1[K1] THEN GOTO M2 ELSE IF T1[K2]>T1[K3]
THEN GOTO M2 ELSE GOTO M1;
P5: C:=0; SEIZE KAHAЛ [K3]; GOTO P4;
T1[K3]:=EXPONENT (L1[K3]);
SEIZE KAHAЛ [K3]; GOTO P6;
P: C:=0; SEIZE KAHAЛ [K3];
P1: T2[K3]:=EXPONENT (L2[K3]); WAIT T2[K3];
IF B CON A THEN WAIT UNTIL (NO B) DIS (NO A);
T1[K3]:=EXPONENT (L1[K3]);
P2: T3[K3]:=EXPONENT (L3[K3]);
WAIT IF T1[K3]>T3[K3] THEN T3 (K3) ELSE T1 [K3];
IF T1[K3]>T3[K3] THEN GOTO P3 ELSE GOTO P1;
P3: C:=1; RELEASE KAHAЛ [K3]; WAIT T1[K3]; GOTO P;
P4: WAIT UNTIL (NO A) DIS (NO B); T1[K3]:=EXPONENT (L1 [K3]);
P6: WAIT IF T1[K3]>T1[K2] THEN T1[K2] ELSE
IF T1[K3]>T1[K1] THEN T1[K1] ELSE T1[K3];
IF T1[K3]>T1[K2] THEN GOTO P2 ELSE
IF T1[K3]>T1[K1] THEN GOTO P2
ELSE GOTO P1 END END.
```

Key: (1). Channel.

Page 156.

Model 3 is constructed on the basis of the general-purpose model of the redundancy of communications. It tracks the steady functioning of the communication network of arbitrary configuration in the assigned time interval of simulation from $t^{\text{мод}}_{\text{нач}}$ to $t^{\text{мод}}_{\text{кон}}$ and puts out the probability of the absence of the readiness of the communication channels. The model the block diagram of algorithm of which is given in figure 5, begins to work from unit 1. This unit, analyzing preset structure, capacities and the versions of the organization of the redundancy of connections from a number of those solved (1, 1/1 or 2/1), and also the parameters of channels, organizes working files and adjusts units 2 and 3. Channel statistics in all directions is fixed/recorded with units 4, 6, 8. Unit 11 establishes/installs the moment/torque of the termination of simulation.

Model, it is analogous with two firsts, it is written in the language of SLANG-system.

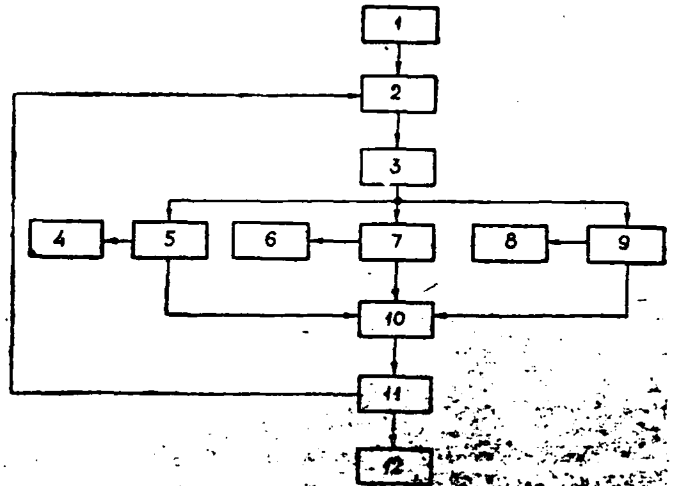

Fig. 5.

Page 157.

Сеть N направлений с R [illegible] (7)
(Резервирование 1, 1/1, 2/1)

```
BEGIN
INTEGER TKOH, K, A1, D, F, F2, HAП[K], PE3[K], T1[K, F],
T3[K, F], T4[K, F], T6[K, F], T7[K, F], T9[K, F];
REAL A2, L1[K, F], L2[K, F], L3[K, F], L4[K, F], L5[K, F]
L6[K, F], L7[K, F], L8[K, F], L9[K, F];
BOOLEAN A[K, F], B[K, F], C[K, F];
FACILITY KAH1[K, F], KAH2[K, F], KAH3[K, F];
INPUT (TKOH, K, F, HAП, PE3, A2, L1, L2, L3, L4, L5, L6, L7, L8, L9);
PROCESS CHAN(K1, F1); INTEGER K1, F1;
BEGIN
NEW CHAN (K1, F1) TO F3; WAIT TKOH; STOP;
F3:
S1: K1:=K1+1; IF K1=K+1 THEN BEGIN A1:=1; GOTO S6;
END; F1:=1; D:=PE3[K1]; S11: GOTO (S2, S3, S4), D;
S2: F2:=HAП[K1]; B[K1, F1]:=FALSE; C[K1, F1]:=TRUE;
NEW CHAN (K1, F1) TO S22; SEIZE KAH2[K1, F1];
WAIT TKOH; CANCEL;
S22:NEW CHAN (K1, F1) TO N11; NEW CHAN (K1, F1) TO S5;
SEIZE KAH3[K1, F1]; WAIT TKOH; CANCEL;
S3: F2:=HAП[K1]/2; B[K1, F1]:=FALSE; C[K1, F1]:=TPUE;
NEW CHAN (K1, F1) TO M11;
NEW CHAN (K1, F1) TO N11; HEN CHAN (K1, F1) TO S5;
SEIZE KAH3[K1, F1]; WAIT TKOH; CANCEL;
S4:F2:=HAП[K1]/3; B[K1, F1]:=TRUE;
C[K1, F1]:=FALSE; GOTO S6;
S5: IF F1=F2 THEN GOTO S1 ELSE F1:=F1+1; GOTO S11;
S6: NEW CHAN (K1, F1) TO M5;
NEW CHAN (K1, F1) TO P5;
NEW CHAN (K1, F1) TO N11;
IF A1=0 THEN NEW CHAN (K1, F1) TO S5; WAIT TKOH;
N11:
A[K1, F1]:=TRUE; T1[K1, F1]:=EXPONENT (L1[K1, F1]);
N: WAIT T1[K1, F1];
A[K1, F1]:=FALSE; SEIZE KAH1[K1, F1];
WAIT EXPONENT (L2[K1, F1]);
IF B[K1, F1] CON C[K1, F1] THEN WAIT UNTIL
(NO B[K1, F1]) DIS (NO C[K1, F1]);
T1[K1, F1]:=EXPONENT (L1[K1, F1]);
T3[K1, F1]:=EXPONENT (L3[K1, F1]);
IF T1[K1, F1] EL T3[K1, F1] THEN GOTO N
ELSE WAIT T3[K1, F1];
A[K1, F1]:=TRUE; RELEASE KAH1[K1, F1]; GOTO N;
M5: T4[K1, F1]:=EXPONENT (L4[K1, F1]);
M: WAIT T4[K1, F1]; B[K1, F1]:=FALSE;
M11: SEIZE KAH2[K1, F1]; IF A1=1 THEN WAIT EXPONENT
(L5[K1, F1]);
M1: T4[K1, F1]:=EXPONENT (L4[K1, F1]);
T6[K1, F1]:=EXPONENT (L6[K1, F1]);
IF A[K1, F1] CON C[K1, F1] THEN WAIT UNTIL
(NO A[K1, F1]) DIS (NO C[K1, F1]);
IF T4[K1, F1] ELF T6[K1, F1] THEN GOTO M
ELSE WAIT T6[K1, F1]; B[K1, F1]:=TRUE;
RELEASE KAH2[K1, F1]; GOTO M;
P5: SEIZE KAH3[K1, F1]; GOTO P11;
P: WAIT T7[K1, F1]; C[K1, F1]:=FALSE;
SEIZE KAH3[K1, F1]; WAIT EXPONENT (L8[K1, F1]);
P11:
IF A[K1, F1] CON C[K1, F1] THEN WAIT UNTIL
(NO A[K1, F1]) DIS (NO B[K1, F1]);
T7[K1, F1]:=EXPONENT (L7[K1, F1]);
T9[K1, F1]:=EXPONENT (L9[K1, F1]);
IF T7[K1, F1] EL T9 [K1, F1] THEN GOTO P
ELSE WAIT T9 [K1, F1]; C[K1, F1]:=TRUE;
RELEASE KAH3[K1, F1]; GOTO P; END END
```

Key: (1). Network/grid N of directions with R channels in each (redundancy 1, 1/1, 2/1).

Page 158.

The models examined were used in the composite model of the network/grid of the exchange of information for the study of the versions of the construction of the network/grid of the computer centers of Kazakh SSR (RSVTs of KazSSR), and also as independent for the preliminary updating of the capacities of communications of the network/grid indicated.

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Page 159.

ANALYSIS OF THE FLEXIBILITY OF INFORMATION-COMPUTING SYSTEMS WITH THE USE/[redacted] OF AN APPARATUS OF THEORY OF GRAPHS.

I. G. Pil'shchikova, A. Kh. Khaybullina, M. M. Sharipova.

During the development of information-computing systems (IVS) is of interest the comparison of the diverse variants of the structures of network/grid. This comparison can be carried out according to the function of flexibility h_{ij}, by understanding under the latter the probability of the existence at least of one path without the loops for any two points of network/grid i, j at the preset values of the probability of the existence (readiness for the work at the given moment/torque) of the channels of communication of network/grid P_{ij}, directly connecting assemblies i, j. In this case each version of network/grid will be characterized by the square matrix/die of flexibility H by size/dimension n, where n - number of assemblies, and h_{ij} — matrix elements, which give the values of the unknown indices.

A question of the determination of the flexibility of structures no longer is new. For the solution of analogous problems was used the apparatus of the systems of mass maintenance/servicing. However, for the case of the evaluation of the structure of the network/grid of the computer centers the apparatus indicated is not suitable in view of multi-linearity and multiphasic nature of the object in question.

The evaluation of the flexibility of structures with the use of logical functions requires the compositions of the ideal normal disjunctive form with the subsequent minimization before obtaining of the simplest disjunctive normal expression. Further transition from the logical functions to the events makes it possible to determine the probability of the unknown event by the methods of the probability theory.

Latter/last method can give accurate result, for the systems, which have only consecutive or parallel connections, and in such a case, when one and the same circuit can be connected with more than one path (for example, bridge circuit), are necessary further conversions for the purpose of obtaining at least approximate estimates for the objective function.

Page 160.

Discussion deals with the representation of system with the complicated connection of elements/cells with the help of the systems only with consecutive and parallel connections [3]. For our case most acceptable is the computational method, based on the theorem about the full/total/complete disintegration of network/grid, which is the generalization of Shannon-Moore theorem [2].

On the basis of this theorem h_{ij} it is obtained as the element function of the network/grid:

$$h_{ij}(P_k)=P_1P_2\ldots P_m\xi_1+P_1P_2\ldots P_{m-1}(1-P_m)\xi_2+\ldots+$$
$$+(1-P_1)(1-P_2)\ldots(1-P_m)\xi_{2^m}.$$

Structure of IVS we represent by graph/count G(v,u), where v - many apexes/vertexes with a power of n, u - many edges/fins with a power of m. Graph/count G allows/assumes parallel edges/fins, but loops it excludes as not having sense for exchange system by information. The edges/fins of graph/count (connection) are characterized by the readiness factors of channels; therefore graphs G can be considered as probabilistic three-dimensional/space (in general) graph/count with weight functions of edges/fins. The topology of network/grid is preset analytically in the form of matrix/die of contiguity M assemblies - assemblies, weighed according to a number of channels.

Matrix element $\mu_{ij}=\begin{cases} k, \text{ if i and j are connected with k- channels} \\ 0, \text{ if assemblies i and j are not connected.}\end{cases}$

Algorithm of the formation of the unknown matrix/die H following.

1. On matrix/die M we construct skeleton matrix/die M* of graph/count, we simultaneously produce contraction of parallel edges/fins.

2. For purpose of decrease of dimension of network/grid we realize equivalent conversions: a) exception/elimination of irredundant edges/fins; b) to roll of consecutive edges/fins with simultaneous contraction of parallel ones, which appear as a result of consecutive convolution.

3. If dimension of network/grid is great, then its further conversion is produced either by method of growth or by method of cut.

4. If dimension of network/grid is small, then calculation of flexibility h_{ij} occurs according to expansion theorem with subsequent return to initial network/grid.

Some transformations of matrix/die M and calculation of flexibility h_{ij} are produced by another form, than in work [4], that economizes the computer memory.

Page 161.

1. during calculation h_{ij} it is examined subgraphs from elements/cells with probability P_i. If it is connected, then expression $\prod_{s,t} P_s(1-P_t)$ (this means that the extended elements/cells have a probability P_t) it enters composed in h_{ij}, i.e. function $\xi=1$. Furthermore, the algorithm of program is limited to the examination of the existence of the paths, which encompass not more than two extended elements/cells, i.e., the calculation of flexibility is performed with an accuracy to thousanths.

2. Roll of consecutive edges/fins it is carried out as follows: if apex/vertex i is directly connected only with two apexes/vertexes of graph/count k, 1, then it is excluded, and probability P_{kl} is computed as

$$P_{kl}=1-(1-P_{kl})(1-P_{ki}P_{il}).$$

File β_1 is the list not only of the excluded, but also boundary apexes/vertexes (in our example this k and $\bar{l}$): if β [s] =i, then β [s+1] =k, β [s+2] $=l$, which facilitates calculations with the reset to the initial network/grid.

Algorithm made it possible to produce the calculation of flexibility h_{ij} for IVS, which consists of 17 apexes/vertexes. Program is comprised in the language ALGOL-60. Scope of the program of the calculation of 2536 nuclei. Program allows, without bringing about a change, to produce calculation for the structure of IVS, which contains from 40 to 160 apexes/vertexes depending on the special features/peculiarities of the structures in question.

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Page 162.

TABULAR METHOD OF CALCULATING THE ELEMENTARY FUNCTIONS IN NONPOSITIONAL NUMERATION SYSTEMS.

V. S. Sedov.

Essence of algorithm. The tabular method of calculating the elementary functions in question is based on the following property of comparisons.

For any polynomial

$$P_n(x)=c_0+c_1x+c_2x^2+\ldots+c_nx^n$$

with whole coefficients $c_0, c_1, \ldots, c_n$ from the integral argument x and for any the whole k occurs the comparison

$$P_n(x+kp)\equiv P_n(x) \pmod p \qquad (1)$$

with any basis/base p.

In the nonpositional numeration system number x is represented in the form of the set of deductions on bases/bases $p_1, p_2, \ldots, p_n$ of operating range P, therefore, if the values of polynomial $P_n(x)$ lie/rest in the range P, polynomial $P_n(x)$ can be presented in the form of the set of the tables of conformity (recodings) between the deductions of argument x and the deductions of the values of polynomial $P_n(x)$ on each basis/base separately.

Number $x: 0 \leqslant x \leqslant 1$ in the mode/conditions of fixed point is represented in the form

$$x = \frac{X}{P}, \qquad (2)$$

where $X \in |\cdot|_P^+ (|\cdot|_P^+$ designates the full/total/complete set of the least non-negative residue on modulus/module $P)$.

Always it is possible to unambiguously present number x in the following form:

$$x = \frac{X_2}{P_2} + \frac{1}{P_2} \cdot \frac{X_1}{P_1}, \qquad (3)$$

where $P_1 \cdot P_2 = P$,

$$X_2 \in |\cdot|_{P_2}^+, \; X_1 \in |\cdot|_{P_1}^+.$$

Page 163.

If x - some fixed value of the argument of function f(x), then, by writing/recording x in the form (3), we will obtain

$$f(x) = f\left(\frac{X_2}{P_2} + \frac{1}{P_2} \cdot \frac{X_1}{P_1}\right) = f_{X_2}(X_1). \qquad (4)$$

Considering value of X_2 as certain parameter, which defines the number of function $f_{X_2}(X_1)$, and X_1 - as the argument of function

$f_{X_2}(X_1)$, it is necessary to each value of argument x of function f(x) to set in the conformity function $f_{X_2}(X_1)$ and value of argument X_1. The value of parameter X_2, which is determining the number of function, and argument X_1 easily they are computed:

$$X_2=\left[\frac{X}{P_1}\right], \quad (5)$$

$$X_1=|X|^+_{P_1}. \quad (6)$$

Thus, the range of change of argument x of function f(x) is divided/marked off into P_2 intervals, moreover the number of interval is equal to X_2. In each interval with number X_2 function f(x) is approximated by function $f_{X_2}(X_1)$.

Taking into account the property, described above, by the best, from the point of view of the nonpositional numeration system, is the representation of function $f_{X_2}(X_1)$, approximating elementary function f(x) in the interval with number X_2, and in the form

$$f_{X_2}(X_1)=\frac{P_n^{(X_2)}(X_1)}{PQ}. \quad (7)$$

where Q - certain further range, which consists of the pair-wise mutually simple between themselves and with the bases/bases of operating range P bases/bases q_j $(j=1, 2, \ldots, m)$; $P_n^{(X_2)}(X_1)$ - the polynomial of the n order with whole coefficients $c_i^{(X_2)}$ from the integral argument X_1:

$$P_n^{(X_2)}(X_1)=c_0^{(X_2)}+c_1^{(X_2)}\cdot X_1+c_2^{(X_2)}\cdot X_1^2+\ldots+c_n^{(X_2)}\cdot X_1^n. \quad (8)$$

Page 164.

We will seek this polynomial of form (8) with the whole coefficients, which would satisfy the following two conditions:

$$1)\qquad \left| f_{X_2}(X_1) - \frac{P_n^{(X_2)}(X_1)}{PQ} \right| < \varepsilon_{\text{зад}}, \qquad (9)$$

where $\varepsilon_{\text{зад}}$ — preset error;

$$2)\qquad 0 \leqslant P_n^{(X_2)}(X_1) < PQ. \qquad (10)$$

Let us rewrite formula (7) in this form:

$$f_{X_2}(X_1) = \sum_{k=0}^{n} a_k^{(X_2)} \cdot X_1^k, \qquad (11)$$

where $a_k^{(X_2)}$ — rational coefficients with the preset denominator PQ.

Let us expand function f(x) in the interval with number X_2 at point X_2/P in the power series according to degrees of X_1 and, being limited n+1 to term of expansion, let us register:

$$f(x)\Big|_{x=\frac{X_2}{P_2}} \sim \sum_{k=0}^{n} \alpha_k^{(X_2)} \cdot X_1^k, \qquad (12)$$

where $\alpha_k^{(X_2)}$ — some real coefficients. Transition from the disintegration with real coefficients (12) to the disintegration with the rational coefficients with the preset denominator PQ (11), or, which is the same, the determination of whole coefficients $c_k^{(X_2)}$ of

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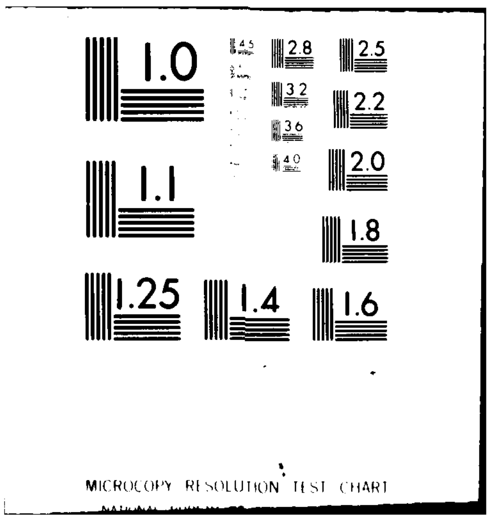
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MICROCOPY RESOLUTION TEST CHART

polynomial (8) can be realized on the following diagram: if

$$\left.\begin{array}{lr}
(1)\ \text{то} & \left\{\alpha_n(x_1)\cdot\frac{PQ}{P_1^n}\right\}<\frac{1}{2}, \\
 & c_k(x_1)=\left[\alpha_k(x_1)\cdot\frac{PQ}{P_1^k}\right]+1, \\
 & (0\leqslant k\leqslant n-1), \\
(2)\ \text{если} & c_n(x_1)=\left[\alpha_n(x_1)\cdot\frac{PQ}{P_1^n}\right]; \\
 & \left\{\alpha_n(x_1)\cdot\frac{PQ}{P_1^n}\right\}>\frac{1}{2}, \\
\text{то}\ (1) & c_k(x_1)=\left[\alpha_k(x_1)\cdot\frac{PQ}{P_1^k}\right], \\
 & (0\leqslant k\leqslant n-1), \\
 & c_n(x_1)=\left[\alpha_n(x_1)\cdot\frac{PQ}{P_1^n}\right]+1.
\end{array}\right| \qquad (13)$$

Key: (1). then. (2). if.

Page 165.

In this case the error δ, which appears upon transfer from the disintegration with the real coefficients for disintegration with the rational coefficients with the preset denominator PQ, which let us name an error in the integral approximation, can be considered on top according to the formula

$$\delta<\frac{(P_1-1)^n}{2PQ}. \qquad (14)$$

General/common/total error $\Delta_{общ}$, appearing during the calculation of elementary functions in the nonpositional numeration system by tabular method, consists of three forms of errors.

1. Error in approximation Δ_a appears during replacement of elementary function f(x) exponential next to finite number of terms.

2. Error in integral approximation δ was introduced above.

Let us designate through δ_{X_2} error in integral approximation for interval with number X_2. Then

$$\delta_{X_2}=\max_{X_1}\left|\sum_{k=0}^{n}a_k^{(X_2)}\cdot X_1^k-\sum_{k=0}^{n}c_k^{(X_2)}\cdot X_1^k\right|.$$

where as δ it is necessary to take maximum error from set $\{\delta_{X_2}\}$, $(0\leqslant X_2<P_2)$:

$$\delta=\max_{X_2}\{\delta_{X_2}\}$$

or

$$\delta=\max_{X_2}\max_{X_1}\left|\sum_{k=0}^{n}a_k^{(X_2)}\cdot X_1^k-\sum_{k=0}^{n}c_k^{(X_2)}\cdot X_1^k\right|. \qquad (15)$$

Revealing formula (15), we will obtain

$$\delta=\max_{X_2}\max_{X_1}\Big|\left(a_0^{(X_2)}\cdot PQ-c_0^{(X_2)}\right)+\left(a_1^{(X_2)}\cdot\frac{PQ}{P_1}-\right.$$
$$\left.-c_1^{(X_2)}\right)\cdot X_1+\left(a_2^{(X_2)}\cdot\frac{PQ}{P_1^2}-c_2^{(X_2)}\right)\cdot X_1^2+ \qquad (16)$$
$$+\ldots+\left(a_n^{(X_2)}\cdot\frac{PQ}{P_1^n}-c_n^{(X_2)}\right)\cdot X_1^n\Big|.$$

3. Rounding off error $\Delta_{окр}$ connected with transition from range PQ to operating range P.

Page 166.

Since all three forms of errors are independent variables, then

$$\Delta_{\text{общ}} = \Delta_{\text{н}} + \delta + \Delta_{\text{окр}}. \qquad (17)$$

Let us consider an error in the integral approximation δ. Let us introduce the designations

$$c_k^{(X_2)} = \left\{ a_k^{(X_2)} \cdot \frac{PQ}{P_1^k} \right\}, \qquad (18)$$

$$b_k^{(X_2)} = \left[a_k^{(X_2)} \cdot \frac{PQ}{P_1^k} \right]. \qquad (19)$$

Then it is possible to register

$$a_k^{(X_2)} \cdot \frac{PQ}{P_1^k} - c_k^{(X_2)} = c_k^{(X_2)} + r_k^{(X_2)}, \qquad (20)$$

where $r_k^{(X_2)} = b_k^{(X_2)} - c_k^{(X_2)}$ — certain integers.

Let us now rewrite formula (16) in the following form:

$$\delta = \frac{1}{PQ} \max_{X_2} \max_{X_1} \left| \sum_{k=0}^{n} (c_k^{(X_2)} + r_k^{(X_2)}) \cdot X_1^k \right|. \qquad (21)$$

Selecting values $r_k^{(X_2)}$ in such a way that δ would take the minimum value, we will obtain best expansion $f_{X_2}(X_1)$.

It is not difficult to obtain the algorithm of the determination of optimum values $r_s^{(x_s)}$, but in view of the unwieldiness of calculations we will not it examine. One should only note that an error in the integral approximation in this case in comparison with the algorithm of the calculations of coefficients $c_s^{(x_s)}$ in diagram (13) in the case of first-order polynomials will be less 2 times, and in the case of the polynomial of the second order - 8 times.

Example of the expansion of functions sin t and cos t. Disintegration is produced on the Chebyshev polynomials. We have

$$\sin kx = 2\sum_{i=0}^{\infty}(-1)^i I_{2i+1}(k)T_{2i+1}(x), \quad (22)$$

$$\cos kx = I_0(k) + 2\sum_{i=1}^{\infty}(-1)^i I_{2i}(k)T_{2i}(x), \quad (23)$$

$$(-1\leqslant x\leqslant 1),$$

where $I_i(k)$ - Bessel function of the first order i-th of order; $T_i(x)$ - Chebyshev polynomials i-th order.

Page 167.

In formulas (22) and (23) let us replace the variable/alternating:

$$x=2t-1,$$

where

$$0\leqslant t\leqslant 1.$$

Then

$$\sin kx = \sin 2kt\cdot\cos k - \cos 2kt\cdot\sin k = 2\sum_{i=0}^{\infty}(-1)^i I_{2i+1}(k)T^*_{2i+1}(t) = u(t), \tag{24}$$

$$\cos kx = \cos 2kt\cdot\cos k - \sin 2kt\cdot\sin k = I_0(k) + 2\sum_{i=1}^{\infty}(-1)^i I_{2i}(k)T^*_{2i}(t) = v(t), \tag{25}$$

where $T_i^*(t)$— displaced Chebyshev polynomials i-th order.

Let us register equations (24) and (25) in the form of system of equations:

$$\left.\begin{aligned}\cos k\cdot\sin 2kt - \sin k\cdot\cos 2kt &= u(t),\\ \sin k\cdot\sin 2kt + \cos k\cdot\cos 2kt &= v(t).\end{aligned}\right\} \tag{26}$$

Solving this system relative to sin 2kt and cos 2kt, we will obtain:

$$\sin 2kt = \sin k\left[I_0(k) + 2\sum_{i=1}^{\infty}(-1)^i I_{2i}(k)T^*_{2i}(t)\right] + \cos k\cdot 2\sum_{i=0}^{\infty}(-1)^i I_{2i+1}(k)T^*_{2i+1}(t), \tag{27}$$

$$\cos 2kt = \cos k\left[I_0(k) + 2\sum_{i=1}^{\infty}(-1)^i I_{2i}(k)T^*_{2i}(t)\right] - \sin k\cdot 2\sum_{i=0}^{\infty}(-1)^i I_{2i+1}(k)\cdot T^*_{2i+1}(t), \tag{28}$$

$$(0<t<1).$$

Page 168.

Let the argument be changed in the interval [0, h], where h -

any positive number. Then taking into account the specific character of the representation of numbers in the mode/conditions of the fixed point of function it is necessary to present in the form

$$\sin kt, \quad \cos kt,$$

where

$$t = \frac{X}{P}, \quad X \in |\cdot|_P^+.$$

After producing the necessary transformations, we will obtain:

$$\sin k\cdot\frac{X}{P} = \sin z[I_0(k) + 2\sum_{i=1}^{\infty}(-1)^i I_{2i}(k)\cdot T^*_{2i}(t)] + \\ + \cos z\cdot 2\sum_{i=0}^{\infty}(-1)^i\cdot I_{2i+1}(k)\cdot T^*_{2i+1}(t), \tag{29}$$

$$\cos k\cdot\frac{X}{P} = \cos z[I_0(k) + 2\sum_{i=1}^{\infty}(-1)^i I_{2i}(k)\cdot T^*_{2i}(t)] - \\ - \sin z\cdot 2\sum_{i=0}^{\infty}(-1)^i I_{2i+1}(k) T^*_{2i+1}(t), \tag{30}$$

where

$$z = k\frac{2X_2+1}{2P_2},$$

$$k = \frac{k}{2P_2},$$

$$t = \frac{X_1}{P_1}.$$

Limiting these expansions by a finite number of members and by substituting the value the displaced Chebyshev polynomials, it is possible on described above algorithm to switch over to disintegration with the rational coefficients with the preset denominator PQ.

It is possible to consider the necessary value of the further range Q. Let in expansions (29) and (30) we be bounded by the first members taking into account the remainder. Counting that an error in the numerical approximation is determined from the formula

$$\delta=\frac{(P_1-1)^n}{qPQ}, \qquad (31)$$

where $q \gg 1$, and by accepting $\Delta_{\text{осн}}=\frac{1}{P}$, we will obtain

$$Q>\frac{2^{2n+1}(n+1)(P_1-1)^n\cdot P_2^{n+1}}{q(\omega\cdot 2^{2n+1}\cdot(n+1)!P_2^{n+1}-P_1^{n+1})}, \qquad (32)$$

where $\omega=\varepsilon_{\text{зад}}\cdot P-1$, $\varepsilon_{\text{зад}}$ — preset error in the calculations.

Page 169.

Algorithm of the tabular method of calculating the elementary functions in nonpositional system of numeration. The code of argument in the remainders/residues is fed in parallel:

$$x\div(\alpha_1, \alpha_2, \ldots, \alpha_n).$$

1. Is determined number of interval X_2 and value of argument X_1. The value of argument X_1 is determined on the bases/bases of operating range P and further range Q.

2. By number of interval X_2 is determined number of table (recoding).

3. On deductions of number X_1 in the range PQ is determined value of elementary function f(x) of table with number X_2.

4. Value of elementary functicn, obtained in the range PQ, it is reduced (if this necessarily) tc range P. on this the calculations are finished.

RESEARCH OF WEIGHT CHARACTERISTICS IN NONPOSITIONAL SYSTEMS.

N. V. Filippova.

Page 170.

Let us consider the weakly-positional system n of the bases/bases:

$$p_1=x-\xi_1,\ p_2=x-\xi_2,\ \dots,\ p_n=x-\xi_n.$$

Integer N is represented in this system in the form of the polynomial

$$N(x)=a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\dots+a_1x+a_0$$

and in the form of the set of remainders/residues from division of N(x) on $p_k(x)$ (k=1, 2, ..., n)

$$N=(\gamma_1, \gamma_2, \dots, \gamma_n).$$

γ_k - remainder/residue from division of N(x) on $p_k(x)$.

$$\gamma_k=N(\xi_k),\quad (k=1, 2, \dots, n).$$

In article [2] was introduced the important integral characteristic of number N

$$W(N)=\sum_{k=1}^{n}\lambda_k L_k,$$

called weight.

Let us trace a change in this characteristic with an increase in the number. It suffices to consider its change for numbers in the range from 0 to $\left[\frac{P-1}{2}\right]$, since in following half of range, from $\left[\frac{P+1}{2}\right]$ to P-1, the weight of a number is uniquely determined by the weight of a number of first half of range.

Page 171.

Theorem. The weight of number N'=P-1-N in the system of the bases/bases

$$p_1=x-\xi_1,\ p_2=x-\xi_2,\ \ldots,\ p_n=x-\xi_n$$

is equal to a difference in number D and weight of number N.

$$W(N')=D-W(N).$$

Proof. Number N can be represented by the set of remainders/residues from the division of polynomial N(x) on $p_k(x)$ (k=1, 2, ..., n):

$$N=(\gamma_1, \gamma_2, \ldots, \gamma_n).$$

After selecting the polynomial of zero degree N(x)=N, we will obtain the set of remainders/residues (N, N, ..., N). Hence

$$L_k(N)=\left[\frac{N}{p_k}\right],\quad (k=1, 2, \ldots, n).$$

For the number

$$N'=P-1-N$$

$$L_k(N')=\left[\frac{P-1-N}{p_k}\right]=\frac{P}{p_k}-\left[\frac{N+1}{p_k}\right]-E_{p_k}^{N+1},$$

$$E_{p_k}^{N+1}=\begin{cases}1, \overset{(1)}{\text{если}}\ N+1\neq 0 \mod p_k,\\ 0, \overset{(2)}{\text{если}}\ N+1=0 \mod p_k.\end{cases}$$

Key: (1). if.

Let us determine the dependence between $\left[\frac{N+1}{p_k}\right]+E_{p_k}^{N+1}$ and $\left[\frac{N}{p_k}\right]$.

$$1)\quad N+1=0 \bmod p_k,\quad \left[\frac{N+1}{p_k}\right]=\left[\frac{N}{p_k}\right]+1,\ E_{p_k}^{N+1}=0,$$

$$2)\quad N+1\neq 0 \bmod p_k,\quad \left[\frac{N+1}{p_k}\right]=\left[\frac{N}{p_k}\right],\quad E_{p_k}^{N+1}=1,$$

$$\left[\frac{N+1}{p_k}\right]+E_{p_k}^{N+1}=\left[\frac{N}{p_k}\right]+1=L_k+1.$$

Express $L_k(N')$ through L_k:

$$L_k(N')=\frac{P}{p_k}-L_k-1.$$

Let us substitute the obtained expression for $L_k(N')$ into the formula of the determination of weight W(N'):

$$W(N')=\sum_{k=1}^{n}\lambda_k\left(\frac{P}{p_k}-L_k-1\right)=P\sum_{k=1}^{n}\frac{\lambda_k}{p_k}-\sum_{k=1}^{n}\lambda_kL_k-\sum_{k=1}^{n}\lambda_k.$$

Page 172.

Since

$$\sum_{k=1}^{n}\lambda_kL_k=W(N),\ P\sum\frac{\lambda_k}{p_k}=D,$$

but

$$\sum_{k=1}^{n}\lambda_k=0,$$

then

$$W(N')=D-W(N).$$

Consequently, it suffices to study the behavior of weight W(N) for numbers of first half of range.

If number N has minimum weight, then number P-1-N will have a weight, maximum among the weights of numbers from 0 to P-1. If $\min W=-\sigma$, then the weight, maximum in this system, is equal to

$\max W=D+\sigma$.

Determination of value D in a n-point weakly-positional system.

Let us consider an n-point weakly-positional system of the bases/bases:

$$p_1(x)=x-\xi_1,\ p_2(x)=x-\xi_2,\ \ldots,\ p_n(x)=x-\xi_n. \qquad (1)$$

Let us determine the values of derivatives at the nodes: $\xi_1, \xi_2, \ldots, \xi_n$. As is known, numbers in this system can be represented unambiguously in the range from 0 to P(x), where $P(x)=\prod_{i=1}^{n}(x-\xi_i)$.

Derivative on x of P(x) is equal to

$$P'(x)=\sum_{k=1}^{n}\prod_{\substack{j=1\\ j\neq k}}^{n}(x-\xi_j). \qquad (2)$$

Substituting in this formula $x=\xi_k$ we will obtain

$$P'(\xi_k)=\prod_{\substack{j=1\\ j\neq k}}^{n}(\xi_k-\xi_j). \qquad (3)$$

Page 173.

D (predicted upper border of a change in weight W of number

$N=(\gamma_1, \gamma_2, \ldots, \gamma_n))$is defined as least common multiple of the values of derivative of P(x) at nodes $\xi_1, \xi_2, \ldots, \xi_n$

$$D=\text{H. O. K.}\{P'(\xi_k)\}.$$

Let us designate thrcugh C quotient of the division $\prod\limits_{\substack{i,j=1\\ i<j}}^{n}(\xi_i-\xi_j)$ into D.

Then D is registered in the form

$$D=\frac{1}{C}\prod_{\substack{i,j=1\\ i<j}}^{n}(\xi_i-\xi_j) \qquad (4)$$

or

$$D=\frac{1}{C}\begin{vmatrix}1 & 1 & 1\ldots & 1\\ \xi_1 & \xi_2 & \xi_3\ldots & \xi_n\\ \xi_1^2 & \xi_2^2 & \xi_3^2\ldots & \xi_n^2\\ \cdot & \cdot & \cdot\ \cdot & \cdot\\ \xi_1^{n-1} & \xi_2^{n-1} & \xi_3^{n-1}\ldots & \xi_n^{n-1}\end{vmatrix}.$$

Value D can be represented as the value cf Vandermondes's determinant.

With increase in n - number cf bases/bases in system (1) - increase the values of derivatives at the nodes, and also rapidly increases a number of cofactors in product (2), of mutually simple with the values derivatives at other nodes. Therefore with an increase in the number of bases/bases D it becomes considerably more. Let us consider change D with an increase of the number of bases/bases in the system.

Determination of number D in symmetrical weakly-positional ones (2n+1)- point systems.

Will consider systems with the odd number of bases/bases. Let (n+1) node $\xi_{n+1}=0$, and all others be arranged/located symmetrically relative to it:

$$-\xi_1, -\xi_2, \ldots, -\xi_n, 0, \xi_n, \ldots, \xi_2, \xi_1.$$

Let us determine the value of the derivative P(x) on x at node ξ_k, where k≠n+1:

$$P'(\xi_k)=\prod_{\substack{j=1\\ j\neq k}}^{2n+1}(\xi_k-\xi_j)=2\xi_k^2\prod_{\substack{j=1\\ j\neq k}}^{n}(\xi_k^2-\xi_j^2). \qquad (5)$$

Page 174.

Let us now determine value by the derivative P(x) to x at node $x=-\xi_k$:

$$P'(-\xi_k)=\prod_{\substack{j=1\\ j\neq 2n+2-k}}^{2n+1}(-\xi_k-\xi_j)=2\xi_k^2\prod_{\substack{j=1\\ j\neq k}}^{n}(\xi_k^2-\xi_j^2). \qquad (6)$$

The right sides of equations (5) and (6) coincide; therefore

$$P'(\xi_k)=P'(-\xi_k).$$

Let us determine the value of derivative at point $x=\xi_{n+1}=0$.

$$P'(0)=(\prod_{j=1}^{n}\xi_j^2)(-1)^n=(-1)^n\prod_{j=1}^{n}\xi_j^2.$$

If x - even number, then everything ξ_j - odd numbers. If x - odd number, everything ξ_j - even numbers.

Let us consider based on particular examples increase in D with growth (2n+1) - number of bases/bases.

1) $n=1, 2n+1=3;$

$$p_1=x-a,\ p_2=x,\ p_3=x+a;$$

$$D=2a^2;$$

2) $n=2, b>a>0;$

$$p_1=x-b,\ p_2=x-a,\ p_3=x,\ p_4=x+a,\ p_5=x+b;$$

$$P(x)=(x-b)(x-a)(x+a)(x+b)x.$$

$$\left.\begin{aligned} P'(b)&=2b^2(b^2-a^2),\\ P'(a)&=2a^2(a^2-b^2),\\ P'(0)&=a^2b^2,\end{aligned}\right\}\quad \begin{aligned}\lambda_1&=a^2,\\ \lambda_2&=-b^2,\\ \lambda_3&=2(b^2-a^2).\end{aligned}$$

$$D=2a^2b^2(b^2-a^2)\ \text{(1) при условии, что}\ (a, b)=1,$$

$$W=a^2(L_1+L_5)-b^2(L_2+L_4)+2(b^2-a^2)L_3.$$

Key: (1). when.

Smallest possible values a and b for the systems of this form following: a=1, b=3.

$$D=2\cdot 9(9-1)=144.$$

Coefficients λ_i are equal to:

$$\lambda_1=\lambda_5=1,\ \lambda_2=\lambda_4=-9,\ \lambda_3=16.$$

Page 173.

The value of weight in the given system is computed from the formula

$$W=L_1-9L_2+16L_3-9L_4+L_5.$$

Let us determine D for the system in which a=2, b=4:

$$\left.\begin{aligned} P'(4)&=2\cdot16(16-4)=2^7\cdot3,\\ P'(2)&=2\cdot4(4-16)=-2^5\cdot3,\\ P'(0)&=4\cdot16=2^6,\end{aligned}\right\} \begin{aligned}\lambda_1&=1,\\ \lambda_2&=-4,\\ \lambda_3&=6,\end{aligned}$$

$$D=3\cdot2^7=384,$$

$$W=L_1-4L_2+6L_3-4L_4+L_5.$$

It is obvious, for the unsymmetrical five-point systems of value D they will be above 144.

$$3)\ n=3,\ c>b>a>0,\ (a,\ b)=1,\ (a,\ c)=1,\ (b,\ c)=1;$$

$$p_1=x-c,\ p_2=x-b,\ p_3=x-a,\ p_4=x,\ p_5=x+a,\ p_6=x+b,$$
$$p_7=x+c;$$

$$P(x)=(x-c)\,(x-b)\,(x-a)x\,(x+a)\,(x+b)\,(x+c);$$

$$\left.\begin{aligned} P'(c)&=2c^2(c^2-b^2)\,(c^2-a^2)\\ P'(b)&=2b^2(b^2-c^2)\,(b^2-a^2)\\ P'(a)&=2a^2(a^2-c^2)\,(a^2-b^2)\\ P'(0)&=-a^2b^2c^2\end{aligned}\right| \begin{aligned}\lambda_1&=a^2b^2(b^2-a^2)\\ \lambda_2&=-c^2a^2(c^2-a^2)\\ \lambda_3&=b^2c^2(c^2-b^2)\\ \lambda_4&=-2(c^2-b^2)\,(c^2-a^2)(b^2-a^2),\end{aligned}$$

$$D=2a^2b^2c^2(c^2-b^2)\,(c^2-a^2)\,(b^2-a^2).$$

The system of mutually simple bases/bases can be constructed at the following values of a, b, c:

$$c=8,\ b=4,\ a=2.$$

$$\left.\begin{aligned} P'(c)&=2\cdot2^6\cdot(64-16)\,(64-4)=2^{13}\cdot3^2\cdot5\\ P'(b)&=2\cdot2^4\cdot(16-64)\,(16-4)=-2^{11}\cdot3^2\\ P'(a)&=2\cdot2^2\cdot(4-64)\,(4-16)=2^9\cdot3^2\\ P'(0)&=-2^6\cdot2^4\cdot2^2=-2^{12}\end{aligned}\right| \begin{aligned}\lambda_1&=1\\ \lambda_2&=-20\\ \lambda_3&=80\\ \lambda_4&=-90,\end{aligned}$$

$$D=2^{13}\cdot3^2\cdot5=368\,640.$$

let us consider the symmetrical seven-point weakly-positional system in which a=1, b=3, c=9.

Page 176.

$$\left.\begin{aligned} P'(c)&=2\cdot3^4\cdot(3^4-3^2)(3^4-1)=2^8\cdot3^6\cdot5\\ P'(b)&=2\cdot3^2\cdot(3^2-3^4)(3^2-1)=-2^7\cdot3^4\\ P'(a)&=2\cdot(1-3^4)(1-3^2)=2^8\cdot5\\ P'(0)&=3^4\cdot3^2 \end{aligned}\right\}\quad \begin{aligned} \lambda_1&=1\\ \lambda_2&=-90\\ \lambda_3&=729\\ \lambda_4&=-1280, \end{aligned}$$

$$D=2^8\cdot3^6\cdot5=1280\cdot729.$$

Thus, small D in the system of seven bases/bases

$$D=368\ 640,$$

$$\min D_3=2,\ \min D_5=144,\ \min D_7=368\ 640.$$

Determination of value D in symmetrical 2n point weakly-positional systems.

Let us consider the systems of four bases/bases. Systems of the form: $p_1=x-b$, $p_2=x-a$, $p_3=x+a$, $p_4=x+b$. Number x number a and b of different parity.

$$P(x)=(x-b)(x+b)(x-a)(x+a),$$

$$\left.\begin{aligned} P'(b)&=2b(b^2-a^2)\\ P'(a)&=2a(a^2-b^2) \end{aligned}\right\}\quad \begin{aligned} \lambda_1&=a\\ \lambda_2&=-b, \end{aligned}$$

$$D=2ab(b^2-a^2),\ \overset{(1)}{\text{если}}\ (a,b)=1.$$

Key: (1). if.

Smallest odd values b and a: b=3, a=1.

$$D=2\cdot3(9-1)=48.$$

Smallest possible even values b and a: b=4, a=2.

$$\left.\begin{aligned} P'(b)&=2\cdot4(4^2-2^2)=96\\ P'(a)&=2\cdot2(2^2-4^2)=-48 \end{aligned}\right\}\quad \begin{aligned} \lambda_1&=1\\ \lambda_2&=-2, \end{aligned}$$

$$D=8\cdot12=96.$$

Let us consider the unsymmetrical system of the form:

$$p_1=x-a,\ p_2=x,\ p_3=x+a,\ p_4=x+b,$$

$$P(x)=(x-a)x(x+a)(x+b),$$

$$\begin{array}{l|l} P'(a)=2a^2(a+b) & \lambda_1=\frac{1}{2}(b-a) \\ P'(0)=-a^2b & \lambda_2=\frac{(a-b)(a+b)}{2} \\ P'(-a)=2a^2(b-a) & \lambda_3=\frac{b(a+b)}{2} \\ P'(b)=-(b+a)b(b-a) & \lambda_4=a^2. \end{array}$$

Page 17?.

Numbers a+b and b-a - even, therefore, P'(a), P'(-a) and P'(-b) are multiple 4.

$$D=a^2b(b^2-a^2).$$

With a=1 D in the system of this form are 2 times less than D in the system, examined earlier.

In this case the smallest possible values b and a: b=3, a=1.

$$D=3(9-1)=24.$$

In the system of four bases/bases minimum value D is equal to

$$\min D_4=24.$$

Let us consider the system of six bases/bases.

Systems of the form:

$$p_1=x-b,\ p_2=x-a,\ p_3=x,\ p_4=x+a,\ p_5=x+b,\ p_6=x+c.$$

$$P(x)=(x-b)(x-a)x(x+a)(x+b)(x+c).$$

$$\left.\begin{array}{l} P'(b)=(b^2-a^2)2b^2(b+c) \\ P'(a)=(a^2-b^2)2a^2(a+c) \\ P'(0)=a^2b^2c \\ P'(-a)=2a^2(a^2-b^2)(-a+c) \\ P'(-b)=2b^2(b^2-a^2)(-b+c) \\ P'(-c)=(c^2-b^2)(c^2-a^2)c \end{array}\right\}.$$

Numbers a, b, c of identical parity. Consequently, difference and sum of any two values a, b and c are even. Hence it is possible to note the following: P'(b), P'(a), P'(-a), P'(-b) and P'(-c) are multiple 2^4. Therefore D must be multiple 2^4.

$$D=\frac{(c^2-b^2)(c^2-a^2)(b^2-a^2)a^2b^2c}{4}.$$

Page 178.

The smallest values a, b, c with which it is possible to construct the system of mutually simple bases/bases, are equal to: a=1, b=3, c=5. Value D in this case is equal to:

$$D=\frac{(25-9)(25-1)(9-1)\cdot 1\cdot 9\cdot 5}{4}=34\,560.$$

Systems of the form:

$$p_1=x-a,\ p_2=x-b,\ p_3=x-c,\ p_4=x+c,\ p_5=x+b,\ p_6=x+a,$$

$$P(x)=(x-a)(x+a)(x-b)(x+b)(x-c)(x+c),$$

$$\left.\begin{array}{l} P'(c)=2c(c^2-b^2)(c^2-a^2) \\ P'(b)=2b(b^2-c^2)(b^2-a^2) \\ P'(a)=2a(a^2-c^2)(a^2-b^2) \end{array}\right\}.$$

Since each of the given here derivatives is multiple 2^5, then value D is equal to

$$D=\frac{abc(c^2-b^2)(c^2-a^2)(b^2-a^2)}{2}.$$

With a=2, b=4, c=6 can be constructed the system of mutually simple bases/bases. Let us determine D in the given system.

$$\left.\begin{aligned}P'(c)&=2\cdot 6(36-16)(36-4)\\P'(b)&=2\cdot 4(16-36)(16-4)\\P'(a)&=2\cdot 2(4-36)(4-16)\end{aligned}\right\}.$$

$$D=2\cdot 6(36-16)(36-4)=7680.$$

Is obvious, this value small among values D in the systems of six bases.

$$\min D_6=7680.$$

Above were examined systems 3, 4, 5, 6, 7 bases/bases and the smallest values D for them.

$$\min D_3=2,\ \min D_4=24,\ \min D_5=144,\ \min D_6=7680,$$
$$\min D_7=368\ 640.$$

With further increase in the number of bases/bases D, it is doubtless, it will increase more rapidly. Even in systems 6, 7 bases/bases of operation on modulus/module D during the determination of weight W will be carried out above 13-19 binary bits.

Page 179.

Therefore the use/application of the systems in which a number of bases/bases is more than five, is inexpedient.

Weakly-positional systems, which do not contain numbers with the negative weight.

1. Let us try to find such weakly-positional systems in which weight of numbers varies from 0 to D.

In the symmetrical four-point systems

$$p_1=x-b,\ p_2=x-a,\ p_3=x+a,\ p_4=x+b\ (b>a)$$

the formula of weight accepts the form

$$W=aL_1-bL_2+bL_3-aL_4.$$

If we for number N select polynomial N(x) of zero degree, then entire $\gamma_i (i=1, 2, 3, 4)$ will be equal to N.

$$N=(\gamma_1, \gamma_2, \gamma_3, \gamma_4)=(N, N, N, N),$$

hence

$$L_k=\left[\frac{N}{p_k}\right]\ (k=1, 2, 3, 4).$$

Let us substitute values L_k into the formula of the weight

$$W=a\left[\frac{N}{x-b}\right]-b\left[\frac{N}{x-a}\right]+b\left[\frac{N}{x+a}\right]-a\left[\frac{N}{x+b}\right].$$

Smallest among the numbers, which have negative weight, is number N, equal to the second basis/base. For this number value of weight is equal

$$W=a\left[\frac{x-a}{x-b}\right]-b,$$

since

$$L_3=\left[\frac{x-a}{x+a}\right]=0,\quad L_4=\left[\frac{x-a}{x+b}\right]=0.$$

Let us determine values of x at which $W\geqslant 0$.

$$a\left[\frac{x-a}{x-b}\right]-b>0,$$

$$\frac{x-a}{x-b}>\frac{b}{a},$$

$$x-a>\frac{b}{a}(x-b),$$

$$x\leqslant b+a.$$

Page 180.

Basis/base $p_1=x-b\geqslant 3$, hence $x\geqslant b+3$, therefore, $a\geqslant 3$. With $a\geqslant 3$ and with $x\leqslant b+a$ it is possible to construct the system of four bases/bases in which $W\in[0,D]$.

With a<3 the weight of a number takes negative values. For example, in system $p_1=x-3$, $p_2=x-1$, $p_3=x+1$, $p_4=x+3$ with any x W takes negative values. Let x=8, then $p_1=5$, $p_2=7$, $p_3=9$, $p_4=11$ form the system, the weight of numbers in which is changed from -2 to 50 (D=48). An example of the system, which does not have the negative values of weight, is system $p_1=3$, $p_2=5$, $p_3=11$, $p_4=13$; the range of a change in the values of weight from 0 to D=480.

From this example it follows that in the systems, which do not have the negative values of weight, value D is considerably greater than in the systems with the negative values of weight.

Value D is determined from the formula

$$D=2ab(b^2-a^2),$$

but the range of a change in the numbers of the given system

$$P(x)=(x^2-a^2)(x^2-b^2).$$

System does not have the negative values of weight, if $x \leqslant a+b$; therefore

$$P\leqslant ab(b+2a)(a+2b),$$
$$D=2ab(b^2-a^2).$$

In this case of P>D, but not more than in $(1+\frac{3a}{b-a})$ the time, i.e., D is close in their value to P.

2. Will consider now five-sharpened systems in which $W>0$. In the symmetrical five-sharpened/five-turned weakly-positional systems of base they are equal to:

$$p_1=x-b,\ p_2=x-a,\ p_3=x,\ p_4=x+a,\ p_5=x+b\ (b>a).$$

The weight of a number in this system is determined from the formula

$$W=a^2L_1-b^2L_2+2(b^2-a^2)L_3-b^2L_4+a^2L_5.$$

The weight of number N, represented by the zero polynomial $N(x)=N$, is equal to

$$W=a^2\left[\frac{N}{x-b}\right]-b^2\left[\frac{N}{x-a}\right]+2(b^2-a^2)\left[\frac{N}{x}\right]-b^2\left[\frac{N}{x+a}\right]+$$
$$+a^2\left[\frac{N}{x+b}\right].$$

Page 181.

Small from the numbers, which have negative weight, number $N=x-a$. Let us determine the weight of this number:

$$W(x-a)=a^2\left[\frac{N}{x-b}\right]-b^2.$$

Let us find that values of x, at which W is positive.

$$a^2\left[\frac{x-a}{x-b}\right]-b^2>0,$$

$$\frac{x-a}{x-b}>\frac{b^2}{a^2},$$

$$x-a>\frac{b^2}{a^2}(x-b),$$

$$x<\frac{b^2+ab+a^2}{a+b}=a+b-\frac{ab}{a+b}. \quad (7)$$

Since $p_1=x-b$ more or is equal to 3, then

$$x\geqslant b+3. \quad (8).$$

Being congruent/equating the right sides of expressions (7) and (8), we will obtain

$$a-\frac{ab}{a+b}>3$$

or

$$\frac{a^2}{a+b}>3.$$

Let us consider the value of the range of this system.

$$P(x)=(x^2-a^2)(x^2-b^2)x.$$

Since

$$x<a+b-\frac{ab}{a+b},$$

that

$$P<[(a+b)^2+\frac{a^2b^2}{(a+b)^2}-2ab-a^2][(a+b)^2+\frac{a^2b^2}{(a+b)^2}-$$

$$-2ab-b^2]\left[a+b-\frac{ab}{a+b}\right]=\left[b^2+\frac{a^2b^2}{(a+b)^2}\right]\left[a^2+\frac{a^2b^2}{(a+b)^2}\right]\times$$
$$\times\left[a+b-\frac{ab}{a+b}\right]=a^2b^2\left[1+\frac{a^2}{(a+b)^2}\right]\left[1+\frac{b^2}{(a+b)^2}\right]\times$$
$$\times\left[a+b-\frac{ab}{a+b}\right]<2a^2b^2(b^2-a^2)=D,$$

since

$$\left[1+\frac{a^2}{(a+b)^2}\right]\left[1+\frac{b^2}{(a+b)^2}\right]<2(b-a)$$

and

$$a+b-\frac{ab}{a+b}<b+a.$$

Page 182.

Thus, we obtained that P<D. Hence it follows that the use/application of the weakly-positional five-point balanced systems in which the weight is changed only from 0 to D, is inexpedient. It is obvious, with a larger number of bases/bases value D will be much more than the range in question.

Relationships/ratios between the weights of systems and subsystems.

Let us consider the system of the bases/bases

$$p_1=x-\xi_1,\ p_2=x-\xi_2,\ \ldots,\ p_n=x-\xi_n.$$

Range of a change of the numbers in this system from 0 to P(x), where

$$P(x)=\prod_{i=1}^{n}(x-\xi_i).$$

Let us register formula fcr determining of D:

$$D = \text{Н. О. К.}\,\{P'(\xi_k) = \prod_{\substack{i=1\\ i\neq k}}^{n} (\xi_k - \xi_i)\,(k=1, 2, \ldots, n)\},$$

$$D = \frac{1}{c} \prod_{\substack{i<j\\ i,j=1}}^{n} (\xi_i - \xi_j).$$

We will obtain expressicn fcr ccefficients λ_k when L_k in the formula of the weight

$$\lambda_k = D : P'(\xi_k),$$

$$\lambda_k = \frac{(-1)^{k-1}}{c} \prod_{\substack{i<j\\ i,j=1\\ i,j\neq k}}^{n} (\xi_i - \xi_j)\ (k=1, 2, \ldots, n).$$

Page 183.

Let us substitute expression for λ_k into the formula of the weight

$$W = \frac{1}{c} \sum (-1)^{k-1} L_k \prod_{\substack{i<j\\ i,j=1;\ i,j\neq k}}^{n} (\xi_i - \xi_j). \qquad (9)$$

The weight of a number of the subsystem cf bases/bases, which consists of $p_1, p_2, \ldots, p_{n-1}$, will be determined acccrding to the formula

$$W_{1,2,\ldots,n-1} = \frac{1}{c_{1,2,\ldots,n-1}} \sum_{k=1}^{n-1} (-1)^{k-1} L_k \prod_{\substack{i<j\\ i,j=1;\ i,j\neq k}}^{n-1} (\xi_i - \xi_j). \qquad (10)$$

Let us now register the fcrmula of the determination of the weight cf a number in the subsystem of bases/bases $p_2, p_3, \ldots, p_n$:

$$W_{2,3,\ldots,n} = \frac{1}{c_{2,3,\ldots,n}} \sum_{k=2}^{n} (-1)^{k} L_k \prod_{\substack{i<j\\ i,j=2;\ i,j\neq k}}^{n} (\xi_i - \xi_j). \qquad (11)$$

Coefficient with L_1 to fcrmula (9) is represented in the form cf the product:

$$\frac{1}{c}\prod_{\substack{i<j\\ i,j=2}}^{n}(\xi_i-\xi_j)=\left[\frac{1}{c_{1,2,\ldots,n-1}}\prod_{\substack{i<j\\ i,j=2}}^{n-1}(\xi_i-\xi_j)\right]\cdot\frac{c_{1,2,\ldots,n-1}}{c}\prod_{i=2}^{n-1}(\xi_i-\xi_n).$$

Expression in the brackets - coefficient with L_1 in formula (10).

Let us now register coefficient when L_k in formula (9) in the form of the linear combination of coefficients when L_k in formulas (10) and (11):

$$\frac{1}{c}\prod_{\substack{i<j\\ i,j=1;\, i,j\neq k}}^{n}(\xi_i-\xi_j)=\frac{1}{c}\left[\prod_{\substack{i<j\\ i,j=1;\, i,j\neq k}}^{n-1}(\xi_i-\xi_j)\right]\prod_{\substack{i=1\\ i\neq k}}^{n}(\xi_i-\xi_n)=$$

$$=\frac{1}{c}\left[\prod_{\substack{i<j\\ i,j=1;\, i,j\neq k}}^{n-1}(\xi_i-\xi_j)\right](\xi_1-\xi_k+\xi_k-\xi_n)\prod_{\substack{i=2\\ i\neq k}}^{n-1}(\xi_i-\xi_n)=$$

$$=\frac{1}{c}\left[\prod_{\substack{i<j\\ i,j=1;\, i,j\neq k}}^{n-1}(\xi_i-\xi_j)\right]\left[\prod_{i=2}^{n-1}(\xi_i-\xi_n)+(\xi_1-\xi_k)\prod_{\substack{i=2\\ i\neq k}}^{n-1}(\xi_i-\xi_n)\right],$$

$$\frac{1}{c}\prod_{\substack{i<j\\ i,j=1;\, i,j\neq k}}^{n}(\xi_i-\xi_j)=\left[\frac{1}{c_{1,2,\ldots,n-1}}\prod_{\substack{i<j\\ i,j=1;\, i,j\neq k}}^{n-1}(\xi_i-\xi_j)\right]\times$$

$$\times\frac{c_{1,2,\ldots,n-1}}{c}\prod_{i=2}^{n-1}(\xi_i-\xi_n)+\left[\frac{1}{c_{2,3,\ldots,n}}\prod_{\substack{i<j\\ i,j=2;\, i,j\neq k}}^{n}(\xi_i-\xi_j)\right]\times$$

$$\times\frac{c_{2,3,\ldots,n}}{c}\prod_{j=2}^{n-1}(\xi_1-\xi_j).$$

Page 184.

Expression in the brackets in first term - coefficient when L_k in formula (10), the secondly term - coefficient when L_k in formula (11).

We convert coefficient when L_n. in formula (9) to this form:

$$\frac{1}{c}\prod_{i<j}^{n-1}(\xi_i-\xi_j)=\left[\frac{1}{c_{2,3,\ldots,n}}\prod_{\substack{i<j\\ i,j=2}}^{n-1}(\xi_i-\xi_j)\right]\frac{c_{2,3,\ldots,n}}{c}\prod_{j=2}^{n-1}(\xi_1-\xi_j).$$

In the brackets is obtained the coefficient when L_n in formula (11).

Weight in system n bases can be represented in the form of the linear combination of the weights of two subsystems (n-1) of the bases/bases:

$$W=\frac{1}{c}\left[c_{1,2,\ldots,n-1}\prod_{i=2}^{n-1}(\xi_i-\xi_n)W_{1,2,\ldots,n-1}-c_{2,3,\ldots,n}\prod_{j=2}^{n-1}(\xi_1-\xi_j)W_{2,\ldots,n}\right].$$

Since the ordering of system nowhere was specified, it is possible instead of p_1 to take another basis/base p_k, and instead of p_n - any basis/base p_j, except p_k.

Thus weight in system n of bases/bases can be represented in the form of the linear combination of the weights of subsystems, formed by exception/elimination of one of the basis of the system:

$$W=\frac{1}{c}\left[c_{1,2,\ldots,k-1,\,k+1,\ldots,n}\prod_{\substack{i=1\\ i\neq k,j}}^{n}(\xi_i-\xi_k)W_{1,2,\ldots,k-1,\,k+1,\ldots,n}-\right.$$

$$\left.-c_{1,2,\ldots,j-1,\,j+1,\ldots,n}\prod_{\substack{i=1\\ i\neq k,j}}^{n}(\xi_j-\xi_i)W_{1,2,\ldots,j-1,\,j+1,\ldots,n}\right].$$

Page 185.

The weight of a number in the system of two bases/bases p_1 and p_2 is determined from the formula

$$W_{12}=L_1-L_2.$$

Weight of a number in system n of the bases/bases

$$W=\sum_{k=1}^{n}\lambda_k L_k.$$

Express is the weight of a number in system n of the bases/bases through the weights of the subsystems of two bases/bases:

$$W=\lambda_1 L_1+\lambda_2 L_2+\ldots+\lambda_n L_n=\lambda_1 W_{12}+(\lambda_1+\lambda_2)W_{23}+$$
$$+(\lambda_1+\lambda_2+\lambda_3)W_{34}+\ldots+\sum_{k=1}^{n-1}\lambda_k W_{n-1,n}.$$

Latter/last component/term/addend is obtained on the basis of the equality

$$\sum_{k=1}^{n}\lambda_k=0.$$

Using this equality, we will obtain

$$W=\lambda_1 W_{12}+(\lambda_1+\lambda_2)W_{23}+(\lambda_1+\lambda_2+\lambda_3)W_{34}+\ldots$$
$$\ldots-(\lambda_{n-2}+\lambda_{n-1}+\lambda_n)W_{n-3,n-2}-(\lambda_{n-1}+\lambda_n)W_{n-2,n-1}-\lambda_n W_{n-1,n}.$$

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Page 186.

Use of threshold elements/cells in some logic circuits.

B. V. Shevkoplyas, R. Zh. Yerzhanov, Yu. Ye. Chicherin.

As is known, the use/application of threshold elements/cells during the construction of logic circuits in a number of cases ensures the considerable decrease of a total number of usable logic valves/gates, and also is decreased the depth of logical networks/grids [1]. There is a series/row of diagrams on the threshold elements/cells, such, as flip-flops, decoders, adders, shift registers and so forth [2, 3, 4]. However, the possibilities of using the threshold elements/cells in the schematics of automation and computer technology for the purpose of their simplification, increase in the reliability and high speed far are not exhausted.

In the present work are examined the following devices/equipment on the threshold elements/cells: highly stable, that corrects highly stable, the controlled restoring organ/control.

Highly stable device/equipment. Works [5, 6] examine the principles of the construction of the highly stable memory elements, made on the logic circuits AND, OR, NOT and their combinations. Such memory elements work in the straight/direct or reverse unitary code and prove to be very efficient during the construction of some devices/equipment [6]. However, since the known elements/cells are nonsynchronized, during the construction of such devices/equipment into their input circuits are built-in the untying valves/gates of the type AND-NOT/OR-NOT, which leads to a reduction in the high speed and to a considerable increase in the number of logic elements. This deficiency/lack is removed during the use of the proposed diagrams (Fig. 1, 2).

Figure 1 shows the diagram of the synchronized highly stable device/equipment, intended for storing the information, represented in the straight/direct unitary code. Diagram is constructed on the inverting threshold elements/cells (PE1, PE2, ..., PEn), which have threshold T=2 and following weights of inputs: w1=1, w2=2.

Page 187.

Input information ($\bar{x}1$, $\bar{x}2$, ..., $\bar{x}n$), represented in the reverse unitary code, is fed to the "single" input of the corresponding threshold elements/cells. The signal of synchronization Y is fed to each threshold element/cell with weight w1 which has n-1 input with a weight of w2, connected with the outputs/yields (z1, z2, ..., zn) of remaining elements/cells.

In the mode/conditions of storage of information (Y=0) on one of the outputs/yields of device/equipment is preserved logical 1, and on the others - logical 0. This state is stable and does not depend on the values of input informational signals. Is actual/real, logical 1, preserved on one of the outputs/yields of device/equipment, it acts on the input of remaining threshold elements/cells with the weight, equal to threshold, which ensures the appearance of logical ones with 0 at the remaining outputs/yields of the devices/equipment which, in turn, are fed to the input of the unexcited element/cell in question. Input informational signals cannot change the state of the unexcited element/cell, since the corresponding weighted sum lies/rests below the threshold of this element/cell.

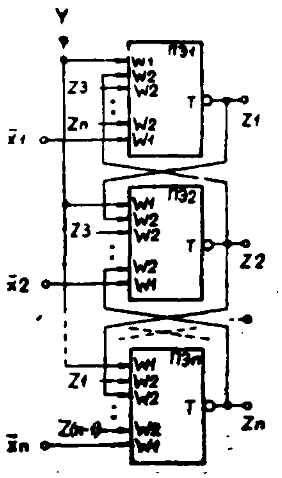

Fig. 1.

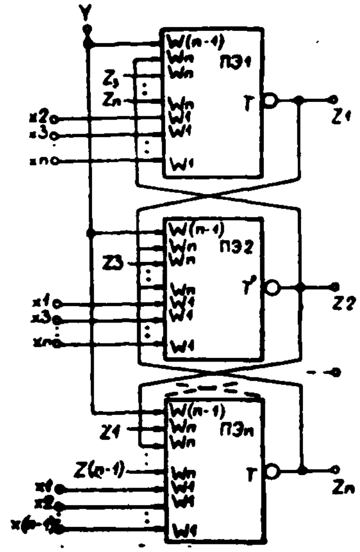

Fig. 2.

Fig. 1. Highly stable device/equipment (first version).

Fig. 2. Highly stable device/equipment (second version).

Page 188.

In the mode/conditions of the recording of information Y=1 the thresholds of all elements/cells seemingly are reduced to units, which ensures the passage of new information to the output/yield of device/equipment. During the supplying of logical to 0 to the synchronizing input Y new information is memorized.

Figure 2 shows the analogous diagram, which accepts the information, represented in the straight/direct unitary code (x1, x2, ..., xn). The weights of the informational input w1=1, of the weight of the input of feedback wn=n, the weight of synchronizing input w(n-1)=n-1, the threshold of element/cell T are equal to the number of input variables n.

Corrective highly stable device/equipment. For the correction of the errors, which appear in the input information, can be used the principle multichannel redundancy, which requires the introduction of special restoring organs/controls [7] to the input of highly stable device/equipment, which leads to an increase in the equipment expenditures and to a reduction in the high speed of this device/equipment. However, in the threshold base this device/equipment, which combines the functions of synchronizing and restoring organs/controls, can be constructed with a minimum number of logic elements (Fig. 3).

Input signals are represented by k channels in each informational direction where k=2l+1, l - multiplicity of correctable errors. All threshold elements/cells (PE1-PEn) have the identical threshold T=2k+1. Weights of the informational input w1=1, the weight of the input of feedback $w(T)=T$, weight of lock inputs $w(l+1)=l+1$.

In the presence of synchronizing impulse/momentum/pulse ($Y=1$) each threshold element/cell realizes a selection of information on the "majority" of input signals. After the termination of synchronizing pulse the corrected information ($z1$, $z2$, ..., zn) is memorized. Thus, is admissible presence lm of the errors in the input information during their even distribution according to the digits.

A similar diagram can be constructed for the correction of information, represented by the straight/direct unitary surplus code. However, in this case considerably increases the number of informational input and rise the thresholds of elements/cells.

Controlled restoring organ/control. The soundness of surplus structures with the restoring organs/controls is monitored with the help of special diagrams [7]. The proposed diagram (Fig. 4), connected to the output/yield of surplus structure, which consists of k of identical devices/equipment, combines renewal functions with the functions of the detection of the errors, which appear in any of logical units.

Page 189.

Diagram contains k noninverting threshold elements/cells PE1, PE2, ..., PEk, to input of which are fed control signals c1, c2 (with weights w1=1 and w(21-1) =21-1; respectively) and the informational signals x1, x2, ..., xk, represented by the direct meanings, with weights w2=2 or w3=3 (Fig. 4). The thresholds of elements/cells are identical and equal to T=21+3.

In the mode/conditions of information retrieval c1=1, c2=0, in this case the device/equipment fulfills the functions of ordinary restoring organ/control [7].

In the mode/conditions of the detection of false zero c1=c2=0, in the mode/conditions of the detection of false unity c1=c2=1, in this case the device/equipment transmits erroneous input one or zero to the appropriate output/yield of device/equipment.

The use/application of similar restoring organs/controls in multistage reserved logical units TsVM [ЦВМ - digital computer] makes it possible to discover and to consecutively/serially" move" an error for any type to the output/yield of device/equipment with the help of the automatically made test programs.

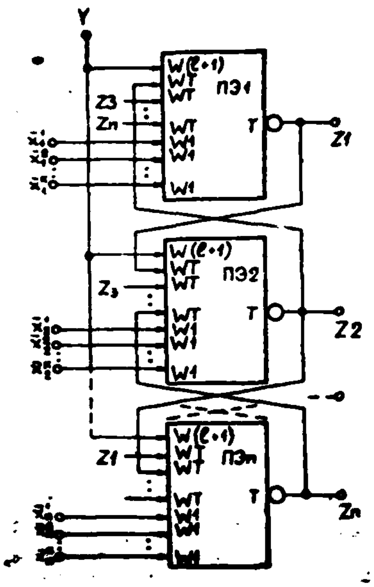

Fig. 3.

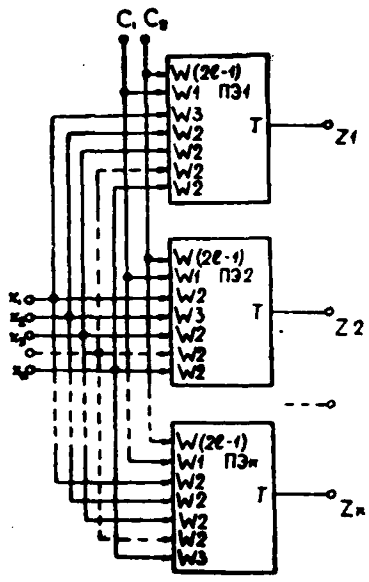

Fig 4.

Fig. 3. Corrective highly stable device/equipment.

Fig. 4. Controlled restoring device.

Page 190.

Comparative evaluations of the expenditures of equipment. The threshold logic elements, utilized in diagrams examined above, can be realized on the basis of resistor-transistor, tunnel- transistor, optical-electronic and other diagrams depending on the required high speed, the required power, freedom from interference, etc. In this case logical comparators, constructed on the uniform components, for example, on the diagrams of DTL and on the threshold elements/cells, which contain diode-resistor linear adder and transistor discriminator. For simplicity let us compare the expenditures of equipment for the analogous diagrams, constructed on resistor-transistor Boolean and threshold elements/cells.

Figure 5 shows the example to the realization of diagram, given in Figure 1, on resistor-transistor threshold elements/cells. Diagram contains 18 resistors and 3 transistors. The analogous diagram, constructed on the elements/cells of the type "logic", contains 21 resistors and 15 transistors. It is possible to show that the diagrams, given in Figures 3 and 4, even more differ significantly from the appropriate diagrams, constructed on the Boolean elements/cells, since in them entirely are used the corrective properties of threshold elements/cells.

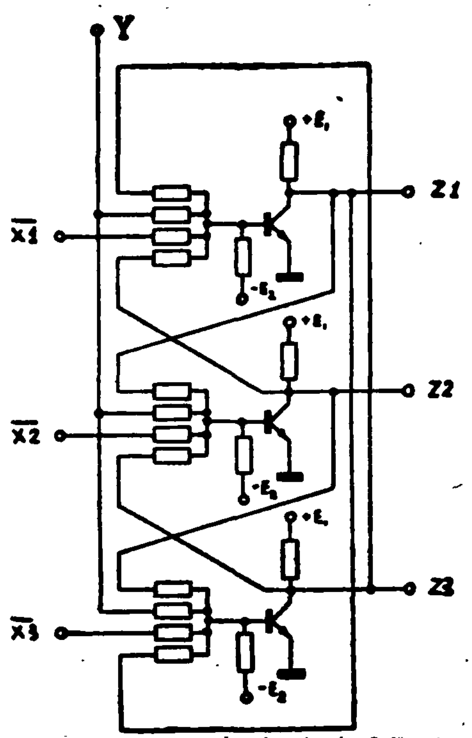

Fig. 5. Example to the realizations of highly stable device/equipment.

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Page 191.

Abstracts.

Pages 3-11.

It is shown that DZU of program with the help of integral technology can be carried out on the same physical basis, as computational units of TsVM. This will lead to an increase in the high speed of special-purpose TsVM.

Illustration 4, References 2.

Pages 12-27.

Article is dedicated to questions of construction and research of different functional dependences taking into account the specific character of deductions concerning modulus/module p, since precisely these questions play principal role in construction and use of systems of nomograms in SSOK.

Illustration 7, References 4.

Pages 28-37.

Article is dedicated to construction and research of nomograms from the adjusted points in the deductions on modulus/module p whose scales and resolving straight lines are constructed in the deductions according to module p. This makes it possible to reduce periodicity on the scales and gives the possibility to use nomograms for the reverse operations.

Illustration 3, References 5.

Page 192.

Pages 38-50.

Is proposed the mathematical model of the process of heating metal in the soaking pits of rolling department. On the basis of the constructed model of object is reproduced one of the used in practice modes/conditions of heating metal in the soaking pit.

Illustration 3, References 14.

Pages 51-64.

In the article are examined the versions of the construction of the self-correcting codes with the use of the finite-difference diagram due to the specific behavior of finite differences in the higher orders and is given generalization of this principle of engineering the self-correcting codes.

References 2.

Pages 65-73.

Is examined the method of the decoding of the nonpositional code, based on obtaining of a difference in the remainders/residues in the control bases/bases with the parallel rounding.

References 8.

Pages 74-79.

In the article is examined the unparallel method of executing the operation of rounding, based on specific ratios between the working and surplus ranges of the representation of numerical information.

References 4.

Page 193.

Pages 80-82.

Article is dedicated to use of one of the heuristic methods of the selection of the structure of the network/grid of the exchange of information in connection with RSVTs of Kazakh SSR.

References 2.

Pages 83-90.

In the article is examined the problem of the optimization of exchange systems by information on stochastic models. For the purpose of the decrease of a quantity of experiments is proposed the algorithm, which varies the principle of search in the process of optimization.

Illustration 1, References 3.

Pages 91-96.

In the article is examined the method, which allows on the mutual location component in the disorder to determine its number, and to also solve inverse problem.

References 3.

Pages 97-103.

Is examined the corrective P(n, k)-code, nonpositional by its nature and provided by positional properties due to the introduction of positional characteristics.

References 3.

Page 194.

Pages 104-114.

In the article is examined the structure of the special-purpose magnetic drum, which makes it possible to realize rapid Fourier transform.

Illustration 7, Table 2, References 2.

Pages 115-119.

Is examined the task of the optimum planning/gliding of the graph of the issue of meltings in the section "steel foundries - the isolation/evolution of soaking pits" metallurgical combine. The quality of planning/gliding is considerad on the statistical model of the section in question.

References 3.

Pages 120-125.

Is realized the separation of many apexes/vertexes of n-dimensional binary cube into the classes of equivalency.

Is described the algorithm of the determination of a number of apexes/vertexes of the classes of equivalency with the use of an apparatus of the theory of generating functions. Is indicated the principle of the single-valued numbering of the elements/cells indicated.

References 5.

Page 195.

Pages 126-139.

Is shown the possibility of using the series/row of the formal procedures of adaptive approach for the solution of the problems of guaranteeing the material and technical supply within the framework of the system of Glavsnab of KazSSR. It is noted that the realization of approach is possible only with the use/application of computers within the framework of ASU of branch.

Illustration 1, References 7.

Pages 140-145.

In the article is examined numeration system, the weight of which is a recurrent sequence of the type of Fibonacci's numbers. Is shown the possibility of the separation of numbers into the groups of digits that, so that during the addition there is no transfer from the group into the group.

Table 2, References 5.

Pages 146-149.

In the article is examined one model of the compression of the information, realized on computers BESM-3M. Model is based on the principle of the place value of storage of digital information.

Illustration 2.

Pages 150-158.

In the article are represented statistical models of communications for exchange systems by information, imitating steady functioning taking into account the diverse variants redundancies. Models are regulated in the increasing complexity. Are given the descriptions of the algorithms of models and their program in the language of SLANG-system.

Illustration 5, Table 1, References 3.

Page 196.

Pages 159-161.

In the article is represented the algorithm of the evaluation of the flexibility of communication network with the parallel edges/fins, using alternate routes of the transmission of information from the source in the discharge.

References 4.

Pages 162-169.

Is given and is traced the algorithm of the calculation of elementary functions with the help of the tables in the nonpositional numeration system.

Pages 170-185.

In the article is examined the change in some weakly-positional systems of the important integral characteristic of a number, called weight, change with an increase in the number of basis of upper bound of the weight of a number, designated by D, and also dependence between the weight of a number in the system and the subsystems.

References 2.

Pages 186-190.

Work examines questions of the construction of some logic circuits on the threshold elements/cells: the synchronized highly stable devices/equipment with the correction and without the correction of errors, restoring organs/controls, which combine the functions of the correction of errors with the functions of check.

Is shown the efficiency of the use of such diagrams in the devices/equipment of automation and computer technology.

Illustration 5, References 7.

Pages 197-198.

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